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Tuk-Tuk: A Unified Account of Similarity Judgment and Analogical Mapping

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Tuk-Tuk: A Unified Account of Similarity Judgment and Analogical Mapping

by

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Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

May 2005

for ultimate freedom

Acknowledgments

I would like to thank Art Markman for his thoughtful and constructive comments at every stage of this dissertation, and for being a great advisor. I am very grateful to Brad Love for taking me under his wing for several years, and for his constant generosity. The other members of my committee, Les Cohen, Bruce Porter, and Brian Stankiewicz, deserve many thanks for diligently reading a long proposal and being very helpful. I would also like to thank Graham Carey and Richard Mitchell, who have played pivotal roles in my education. In addition, I have been very fortunate to receive support as a University Fellow, Computational and Applied Mathematics Fellow, David Bruton, Jr. Graduate Fellow, and National Defense Science and Engineering Graduate Fellow.

Todd Gureckis, Yasu Sakamoto, Patience Henson, Per-Gunnar Martinsson, and Joel Tropp, thank you for your friendship and sharing the experience of graduate school with me. Mom and Dad, in both a literal and profound sense, you have given me my life. Bena, your love and support has made all the difference.

Levi Benjamin Larkey

The University of Texas at Austin

May 2005

Tuk-Tuk: A Unified Account of Similarity Judgment and Analogical Mapping

Publication No. ____________________

Levi Benjamin Larkey, Ph.D.
The University of Texas at Austin, 2005

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While similarity and analogy have traditionally been viewed as involving distinct psychological processes, the thesis of this dissertation is that similarity and analogy invoke the same mapping process. This thesis is supported by a growing body of evidence (Markman & Gentner, 1993a, 1993b, 1996; Gentner & Markman, 1997; Goldstone, 1994; Goldstone & Medin, 1994; Larkey & Markman, 2004). The main contribution of this dissertation is a unified account of similarity judgment and analogical mapping. This account is instantiated as Tuk-Tuk, a localist connectionist model of similarity and analogy that determines a mapping between representations via a dynamic process of interactive activation among feature, object, and relation correspondences. Tuk-Tuk differs from extant models of similarity and analogy in its ability to account for both patterns of similarity ratings and

benchmark phenomena of analogy. In this dissertation, Tuk-Tuk's performance is tested and contrasted with other models using a broad set of simulations, including simulations of a behavioral study of my own design.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Similarity and Analogy in Cognition

Similarity has long been posited as factotum to cognition. According to William James (1890/1950), "This sense of Sameness is the very keel and backbone of our thinking" (p. 459). Fred Attneave (1950) wryly writes that "The question 'What makes things seem alike or seem different?' is one so fundamental to psychology that very few psychologists have been naive enough to ask it" (p. 516).

Numerous theories in cognitive psychology are based on similarity. Similarity provides a basis for generalization (Shepard, 1987). Memory traces are activated according to their similarity to probes (Hintzman, 1986). Objects are categorized according to their similarity to category exemplars (Medin & Schaffer, 1978; Nosofsky, 1986, 1992) or category prototypes (Rosch & Mervis, 1975; Posner & Keele, 1968; Reed, 1972). Decisions may be based on the similarity of the situation that would result from a choice to an ideal situation (Medin, Goldstone, & Markman, 1995). Strategies used to solve previous problems are applied to new problems that are similar (Novick, 1988, 1990; Bassok, 1990; Kolodner, 1993). The strength of an inductive argument depends on the similarity of the target of the argument to the base of the argument (Osherson, Smith, Wilkie, Lopez, &

Shafir, 1990).

Relations, which are extrinsic relationships between objects, and features, which are intrinsic properties of objects, are psychologically distinct (Clement & Gentner, 1991; Gentner, 1988; Gentner & Clement, 1988; Gentner & Landers, 1985; Gentner & Rattermann, 1991; Larkey, Narvaez, & Markman, 2004). For example, common relations and common features function as two different psychological pools when subjects judge perceptual similarity (Goldstone, Gentner, & Medin, 1989; Goldstone, Medin, & Gentner, 1991; Medin, Goldstone, & Gentner, 1990).

Analogy is a kind of similarity where compared items share many relations, but the objects embedded in the relations share few features (Gentner, 1983, 1989). For example, Rutherford's atom is analogous to the solar system because the electrons revolve around the nucleus as the planets revolve around the Sun (Gentner, 1983; Falkenhainer, Forbus, & Gentner, 1989). This analogy is based on the shared "revolves around" relation; few features are shared by the electrons and the planets or the nucleus and the Sun.

Analogy is a critical component of cognitive processing. Even adult chimpanzees (*Pan troglodytes*) have demonstrated analogical reasoning capabilities (Oden, Thompson, & Premack, 2001; Gillan, Premack, & Woodruff, 1981). Analogies are found in the earliest preserved literature. Four thousand years ago, an Egyptian poet (trans. by Merwin, 1968) wrote "Death is before me today/ like the sky when it clears/ like a man's wish to see home after numberless years of captivity" (qtd. in Holyoak, Gentner, & Kokinov, 2001, p. 4). The word "analogy" is derived from the Greek phrase "ana logos" meaning "according to ratio," which was used by Plato in arguing that the idea of the Good makes knowledge possible as the Sun makes vision possible (Plato, trans. 1935, sec. 508c).

Analogies play an important role in the development of scientific theories. Kepler developed new concepts of planetary motion by analogy to the phenomenon of light emanating from the Sun to illuminate the planets (Gentner et al., 1997; Gentner, 2002). Maxwell used mechanical analogies to develop concepts of electromagnetism (Nersessian, 1992). An analogy drawn between light and sound led to the wave theory of light (Holyoak & Thagard, 1995). Detailed observations of the discovery process in microbiology laboratories suggest that research groups that make frequent use of analogies have a creative edge over those that do not (Dunbar, 1995). Perhaps the most often used example of analogy is Rutherford's analogy between the atom and the solar system.

While William James (1890/1950) observed that "a native talent for perceiving analogies is... the leading fact in genius of every order" (p. 530), analogy is not exclusive to genius. Analogies are ubiquitous in our daily cognitive activities. Solutions to analogous problems are applied to novel problems and can yield general problem solving schemas (Gick & Holyoak, 1980; Holyoak, 1984; Bassok, Chase, & Martin, 1998). Analogy abounds in everyday language, as exemplified by phrases such as, "we are at a crossroads," "my job is a jail," and "rumors are weeds" (Lakoff & Johnson, 1980). Analogies help people understand new concepts in terms of more familiar concepts. For example, the Internet company Napster used an analogy between their file downloading software and videocassette recorders to argue that their software should be legal even though it could be used to violate copyright laws. The Internet was introduced as an "information superhighway." Students are taught to think of electricity as analogous to water flowing through pipes (Gentner & Gentner, 1983).

Political rhetoric also makes frequent use of analogy. In the 2004 presidential race, George W. Bush claims "we've turned the corner," while John Kerry's campaign draws an analogy between steering a Swift boat in Vietnam and "setting a new course for America." Others analogize the war in Iraq with the Vietnam War. Analogies that involve transfer of emotions can be very persuasive (Blanchette & Dunbar, 2001; Thagard & Shelley, 2001).

1.2 Thesis and Contribution

At the core of analogy is mapping of mental representations (Hesse, 1966; Gentner, 1983, 1989; Falkenhainer et al., 1989; Holyoak & Thagard, 1989; Keane & Brayshaw, 1988;

Hummel & Holyoak, 1997, 2003; Hofstadter, 1984, 1995). Mapping is the process of determining which elements of one representation correspond to the elements of another representation. For example, in Rutherford's analogy between the atom and the solar system, the nucleus corresponds to the Sun and the electrons correspond to the planets.

Psychological constraints on mapping have been studied extensively with respect to analogy (Markman & Gentner, 2000; Holyoak & Thagard, 1989; Hummel & Holyoak, 2003). While similarity and analogy have traditionally been viewed as involving distinct psychological processes, the thesis of this dissertation is that similarity and analogy invoke the same mapping process. This thesis is supported by a growing body of evidence (Markman & Gentner, 1993a, 1993b, 1996; Gentner & Markman, 1997; Goldstone, 1994; Goldstone & Medin, 1994; Larkey & Markman, 2004). The main contribution of this dissertation is a unified account of similarity judgment and analogical mapping. This account is instantiated as Tuk-Tuk, a localist connectionist model of similarity and analogy that determines a mapping between representations via a dynamic process of interactive activation among feature, object, and relation correspondences. Tuk-Tuk differs from extant models of similarity and analogy in its ability to account for both patterns of similarity ratings and benchmark phenomena of analogy. In this dissertation, Tuk-Tuk's performance is tested and contrasted with other models using a broad set of simulations, including simulations of a behavioral study of my own design.

Chapter 2

Empirical Findings and Benchmark Phenomena

2.1 Similarity Judgment

2.1.1 Structured Representations

Structured representations capture the compositional nature of mental representations by binding predicates to their arguments. The scope of a predicate is limited to its arguments. That is, predicates describe their arguments. For example, a red cape can be represented in predicate notation as RED(CAPE). The scope of the attribute RED is restricted to describing CAPE. An attribute is a predicate that takes one argument. A relation is a predicate that takes two or more arguments. For example, WEARS(SUPERMAN, CAPE) is a relation that represents the knowledge that Superman wears a cape.

Structured representations are critical to represent knowledge unambiguously (Fodor & Pylyshyn, 1988; Biederman, 1985). For example, unstructured representations such as feature sets and multidimensional spaces do not readily capture the difference between a striped square above a shaded circle and a striped circle above a shaded square (Markman,

(a)

(b)	STRIPED	STRIPED
	SHADED	SHADED
	SQUARE	SQUARE
	CIRCLE	CIRCLE
	ABOVE	ABOVE
(c)	STRIPED-SQUARE	STRIPED-CIRCLE
	SHADED-CIRCLE	SHADED-SQUARE
	STRIPED-ON-TOP	STRIPED-ON-TOP
	SHADED-ON-BOTTOM	SHADED-ON-BOTTOM
	SQUARE-ON-TOP	CIRCLE-ON-TOP
	CIRCLE-ON-BOTTOM	SQUARE-ON-BOTTOM
	STRIPED-SQUARE-ON-TOP	STRIPED-CIRCLE-ON-TOP
	SHADED-CIRCLE-ON-BOTTOM	SHADED-SQUARE-ON-BOTTOM
	STRIPED-ABOVE-SHADED	STRIPED-ABOVE-SHADED
	STRIPED-ABOVE-CIRCLE	STRIPED-ABOVE-SQUARE
	SQUARE-ABOVE-SHADED	CIRCLE-ABOVE-SHADED
	SQUARE-ABOVE-CIRCLE	CIRCLE-ABOVE-SQUARE
	STRIPED-SQUARE-ABOVE-SHADED	STRIPED-CIRCLE-ABOVE-SHADED
	STRIPED-SQUARE-ABOVE-CIRCLE	STRIPED-CIRCLE-ABOVE-SQUARE
	STRIPED-ABOVE-SHADED-CIRCLE	STRIPED-ABOVE-SHADED-SQUARE
	SQUARE-ABOVE-SHADED-CIRCLE	CIRCLE-ABOVE-SHADED-SQUARE
	STRIPED-SQUARE-ABOVE-SHADED-CIRCLE	STRIPED-CIRCLE-ABOVE-SHADED-SQUARE
(d)	STRIPED(SQUARE)	STRIPED(CIRCLE)
	SHADED(CIRCLE)	SHADED(SQUARE)
	ABOVE(SQUARE, CIRCLE)	ABOVE(CIRCLE, SQUARE)

Figure 2.1: Two configurations from Markman (1999).

1999). These configurations are shown in Figure 2.1 (a).

An unstructured representation such as the set of simple features shown in Figure 2.1 (b) does not capture the difference between the two configurations because both configurations are represented by the same set of features. The two configurations can be disambiguated by including the conjunctive features shown in Figure 2.1 (c) in addition to the simple features shown in Figure 2.1 (b) (Hayes-Roth & Hayes-Roth, 1977; Gluck, 1991), but such feature sets grow unwieldy even for relatively simple stimuli. For example, representing the two configurations requires 44 simple and conjunctive features. In contrast, the structured representations shown in Figure 2.1 (d) efficiently and unambiguously represent the two configurations.

Spatial representations are even less amenable than feature sets to capturing the structure of mental representations. In addition to the difficulties described above, spatial representations require dimensions that can take values such as striped-above-circle. The meaning of such dimensions can be difficult to interpret. While some stop-gap fixes to feature-set and spatial representations are possible, these fixes fall short of a general capacity to capture the structure of mental representations.

Markman and Gentner (2000) conducted a direct, concentrated study of the role of structure in comparisons. The study demonstrates that similarity processes utilize structured representations. Subjects were shown eight forced-choice triads in random order and were asked to choose which of two target stimuli was most similar to the base stimuli (see Figure 2.2). The stimulus shown in the left column of Figure 2.2 is the base. In all cases, the target shown in the middle column was preferred over the target shown in the right column by a majority of participants as indicated by the numbers to the right of the targets.

Triad 1 verifies that stimuli with similar objects are more similar than stimuli with dissimilar objects. This result is equally compatible with unstructured and structured representations since the commonalities between the preferred target and the base are

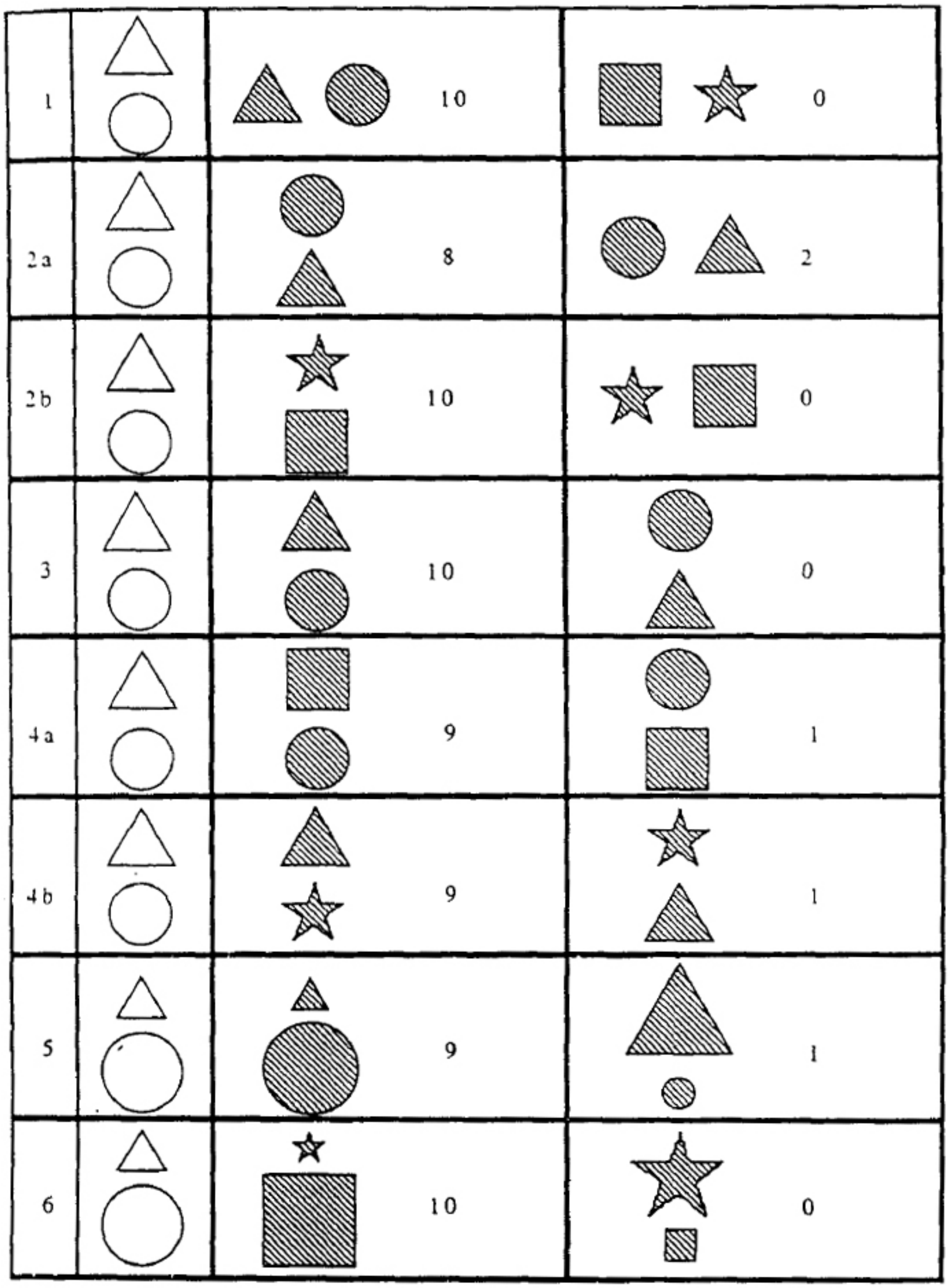

Figure 2.2: Stimuli used by Markman and Gentner (2000) to demonstrate that similarity processes utilize structured representations.

simple features. Triads 2a and 2b demonstrate that stimuli with the same relation are more similar than stimuli without the same relation, even when objects that play the same relational roles are different. In Triad 2a, the preferred target has the same ABOVE relation as the base, but the objects from the base are reversed. In Triad 2b, the shapes in the targets are different from those in the base. Again, this result is compatible with both unstructured and structured representations, as long as we allow the relation ABOVE to be encoded as a simple feature.

An interesting question is which objects in the preferred target in Triad 2b correspond to which objects in the base. According to Gentner's (1983) structure-mapping theory, the triangle corresponds to the star and the circle corresponds to the square because they fill the same roles in the relations ABOVE(TRIANGLE, CIRCLE) and ABOVE(STAR, SQUARE). TRIANGLE and STAR correspond because they are the first arguments to their respective ABOVE relations (i.e., they are the top objects), and CIRCLE and SQUARE correspond because they are the second arguments of their respective ABOVE relations (i.e., they are the bottom objects). This is an example of the structural constraint of parallel connectivity, which requires that the arguments of corresponding predicates themselves be placed into correspondence. Parallel connectivity is discussed in detail in Section 2.2.3; the issue here is that unstructured representations do not support parallel connectivity because they do not capture predicate-argument bindings. These bindings are characteristic of structured representations.

Triad 3 shows that stimuli with similar objects playing the same relational roles are more similar than those with similar objects playing different relational roles. This result demonstrates the need of unstructured representations to include conjunctive features such as TRIANGLE-ABOVE-CIRCLE. Triads 4a and 4b show that stimuli with only one similar object playing the same relational role are more similar than those having no similar objects playing the same relational role. Again, unstructured representations require even more conjunctive features such as CIRCLE-ON-BOTTOM and TRIANGLE-ON-TOP. Triad 5 demonstrates that people prefer consistency across a number of relations in a scene. In the base, the triangle is above the circle, and the triangle is also smaller than the circle. The target that preserves both of these relational commonalities is preferred over the target that preserves only one of them. This result requires additional conjunctive features such as TRIANGLE-SMALLER-THAN-CIRCLE-AND-TRIANGLE-ABOVE-CIRCLE. Triad 6 demonstrates that this preference holds even when the objects are different. To capture this last result with unstructured representations would require additional conjunctive features such as OBJECT1-SMALLER-THAN-OBJECT2-AND-OBJECT1-ABOVE-OBJECT2, but such abstract features are problematic because it is not clear what the placeholders OBJECT1 and OBJECT2 correspond to in a given configuration. At this point, unstructured representations are untenable. In contrast, the base in Triad 6, for example, is efficiently represented using the structured representation SMALLER(TRIANGLE, CIRCLE), ABOVE(TRIANGLE, CIRCLE).

2.1.2 Matches in Place and Matches out of Place

There are two types of commonalities between compared items (Goldstone, 1994). A match in place (MIP) is a match between corresponding elements of compared items. A match out of place (MOP) is a match between elements that do not correspond. For example, when comparing a bird with a grey head and red wings to a bird with a grey head and a red tail, the colors of the birds' heads constitute a MIP because the heads correspond, whereas the red wings and the red tail are a MOP because the wings and tail do not correspond.

A study by Goldstone (1994) demonstrates that MIPs have a greater influence on similarity than MOPs. Subjects rated the similarity between two pairs of butterflies using a scale from one (low similarity) to nine (high similarity). The butterflies varied on four dimensions: head type, tail type, body shading, and wing shading. The pairs were designed such that the number of MIPs and MOPs were independently varied.

On each trial, a base pair was randomly constructed and a target pair was constructed by selectively changing feature dimensions of the base pair in one of six ways, as

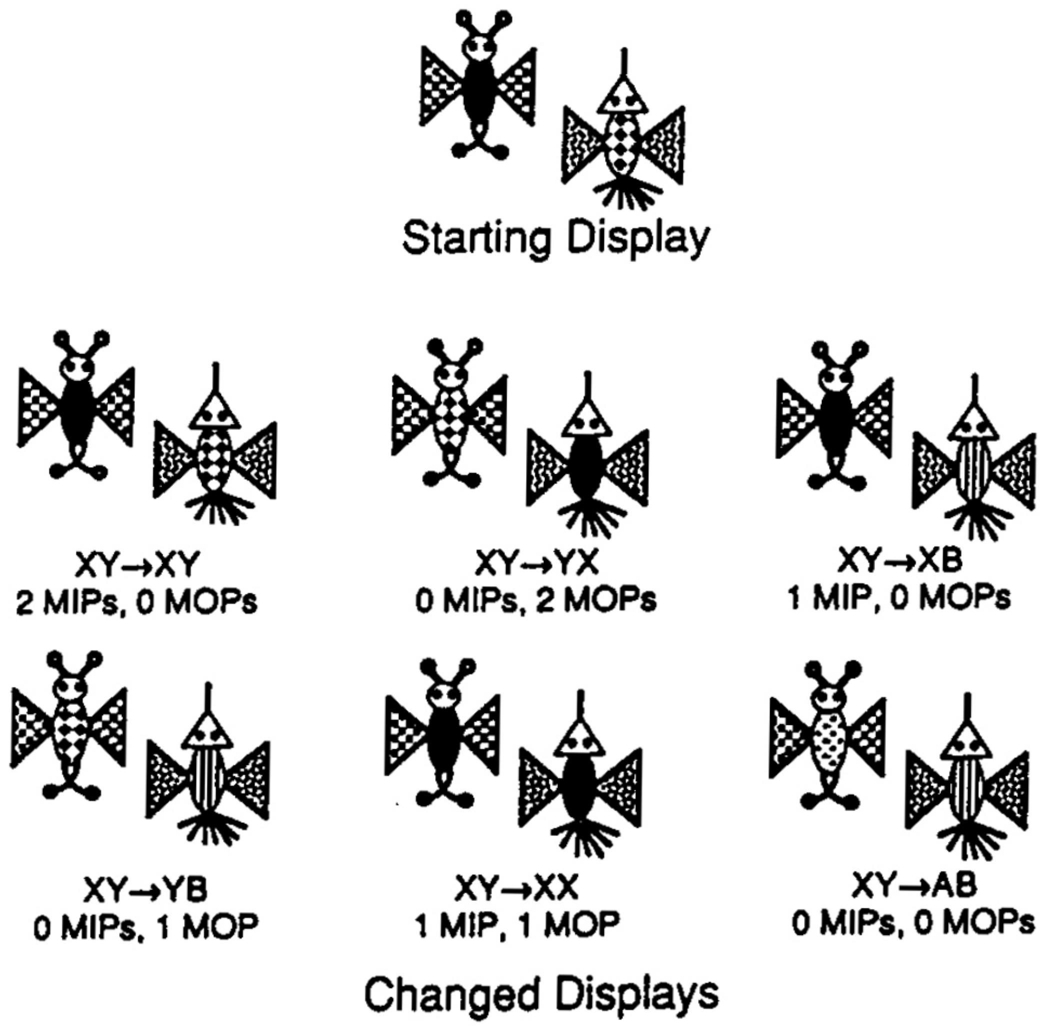

Figure 2.3: Stimuli used by Goldstone (1994) to demonstrate that MIPs have a greater influence on similarity than MOPs.

illustrated in Figure 2.3 (from Goldstone, 1998). A pair's respective values on a particular dimension can be abstractly represented by letters. For example, a pair with body shading represented by XY has body shading X for one butterfly and body shading Y for the other butterfly. If XY denotes the base pair's respective values on a particular dimension, then the methods used to change a feature dimension are: XY→XY (no change, 2 MIPs and 0 MOPs), XY→YX (switch values, 0 MIPs and 2 MOPs), XY→XB (replace one value, 1 MIP and 0 MOPs), XY→YB (replace one value and switch values, 0 MIPs and 1 MOP), XY→XX (copy one value, 1 MIP and 1 MOP), XY→AB (replace both values, 0 MIPs and 0 MOPs).

The mean similarity ratings for 0, 1, and 2 MIPs were 5.5, 6.4, and 7.1, respectively. The mean ratings for 0, 1, and 2 MOPs were 5.5, 5.5, and 5.9, respectively. Similarity increases with MOPs as well as MIPs, but MIPs increase similarity to a greater extent than MOPs. This finding is also demonstrated in patterns of similarity judgments from an experiment of my own design, which is described in detail in Section 5.1.

2.1.3 Alignable and Nonalignable Differences

Like MIPs versus MOPs, there are two types of differences between compared items (Markman & Gentner, 1993a). Alignable differences are differences between corresponding elements of compared items. For example, an alignable difference between a car and a motorcycle is the number of wheels they have. Nonalignable differences are differences between elements that do not correspond or differences where an element in one representation does not correspond to any element in the other representation. For example, a seat belt is a nonalignable difference between a car and a motorcycle because a motorcycle has no restraining device that corresponds to a car's seat belt. Alignable differences and nonalignable differences are psychologically distinct. Similar items tend to have more alignable differences than dissimilar items (Markman & Gentner, 1993a). Alignable differences are easier to list, serve as better memory probes, and have a greater influence on similarity

Similar pairs		Dissimilar pairs	
Yacht	Sailboat	Curtain	Ball Bearing
Hotel	Motel	Eggplant	Giraffe
VCR	Tape Deck	Restaurant	Strobe Light
Kite	Hang Glider	Bank Check	Light Bulb
Broom	Mop	Magazine	Kitten
Watch	Clock	Mug	Speaker
Ice Cream Sundae	Banana Split	Phone Book	Lamp Shade
Sculpture	Painting	Postage Stamp	Microphone
Police Car	Ambulance	Lock	Asphalt
Rocket	Missile	Blanket	Bowl
Chair	Stool	Cab Driver	Antenna
Calculator	Abacus	Air Conditioner	Cloud
Army	Navy	Door	Sidewalk
Bed	Couch	Stove	Dumpster
Casino	Horse Track	Notebook	Piano
Store	Boutique	Traffic Light	Shopping Mall
Hammock	Lounge Chair	Freezer	Personal Computer
Stairs	Escalator	Trapeze	Fork
McDonald's	Burger King	Parade	Tennis
Football	Hockey	Handcuffs	T-Shirt

Table 2.1: Word pairs used by Markman and Gentner (1993a).

than nonalignable differences (Markman & Gentner, 1993a, 1996, 1997).

To test whether similar items tend to have more alignable differences than dissimilar items and whether alignable differences are easier to list, Markman and Gentner (1993a) had subjects list commonalities and differences of highly similar word pairs and highly dissimilar word pairs (see Table 2.1). More commonalities and alignable differences were listed for similar pairs than for dissimilar pairs. This result supports the view that systems of commonalities that facilitate similarity also raise the salience of differences that are conceptually related to those commonalities. In addition, more alignable differences than nonalignable differences were listed overall. This result supports the view that differences related to commonalities (i.e., alignable differences) are more salient than unrelated differences (i.e., nonalignable differences).

A study by Markman and Gentner (1997) demonstrates the effects of alignability

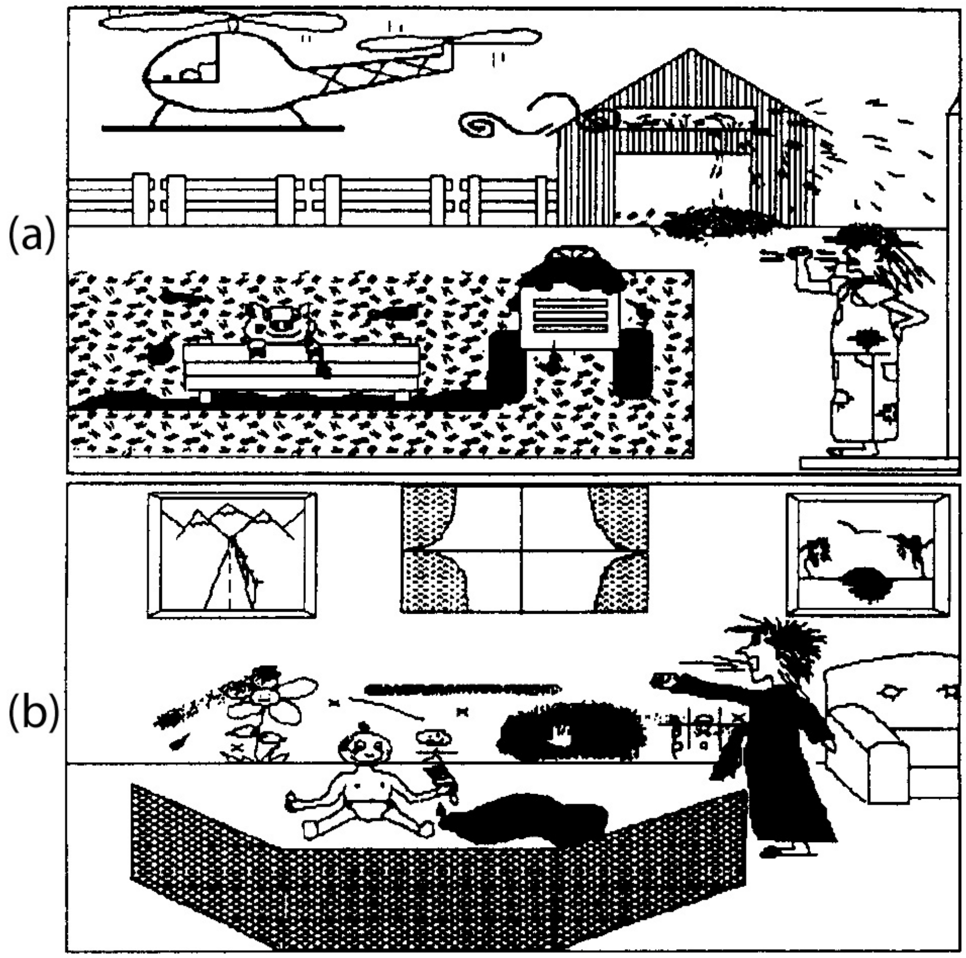

Figure 2.4: Two scenes used by Markman and Gentner (1997) to demonstrate that alignable differences are better memory probes than nonalignable differences.

on memory. Subjects compared pairs of pictures and then were probed for recall. For example, subjects rated the similarity between the scene shown in Figure 2.4 (a) and the scene shown in Figure 2.4 (b). After a 30-minute delay, subjects were shown an item taken from one of the pictures. The item was either an alignable difference (e.g., the pig from Figure 2.4 (a)) or a nonalignable difference (e.g., the helicopter from Figure 2.4 (a)). The subject was asked to recall as much as possible about the scene from which the cue came. The central result is that alignable differences are better memory probes than nonalignable differences, which suggests that people attend to corresponding information more than non-corresponding information when making comparisons.

An earlier study by Markman and Gentner (1996) demonstrates that variations in alignable differences affect similarity more than variations in nonalignable differences. Subjects submitted similarity ratings for eight sets of four pairs like the set in Figure 2.5. Stimulus A is paired separately with each of stimuli B, C, D, and E. A car versus a truck being fixed is an alignable difference in the first pair (A with B). In the second pair (A with C), a car versus a robot being fixed is also an alignable difference, but the difference is greater than a car versus a truck being fixed because a car and a robot are more dissimilar than a car and a truck. In D and E, the same two items (a truck or a robot, respectively) are nonalignable differences added as some other item on the floor. The central result is that the variation of the item matters much more for alignable differences than for nonalignable differences. That is, when subjects rate the similarity of all four pairings of the base (A) with a target (B, C, D, or E), there is a greater difference in rated similarity between the two alignable difference pairs (A with B and A with C) than between the two nonalignable difference pairs (A with D and A with E). Estes and Hasson (2004) have replicated this finding using simple geometric configurations as stimuli.

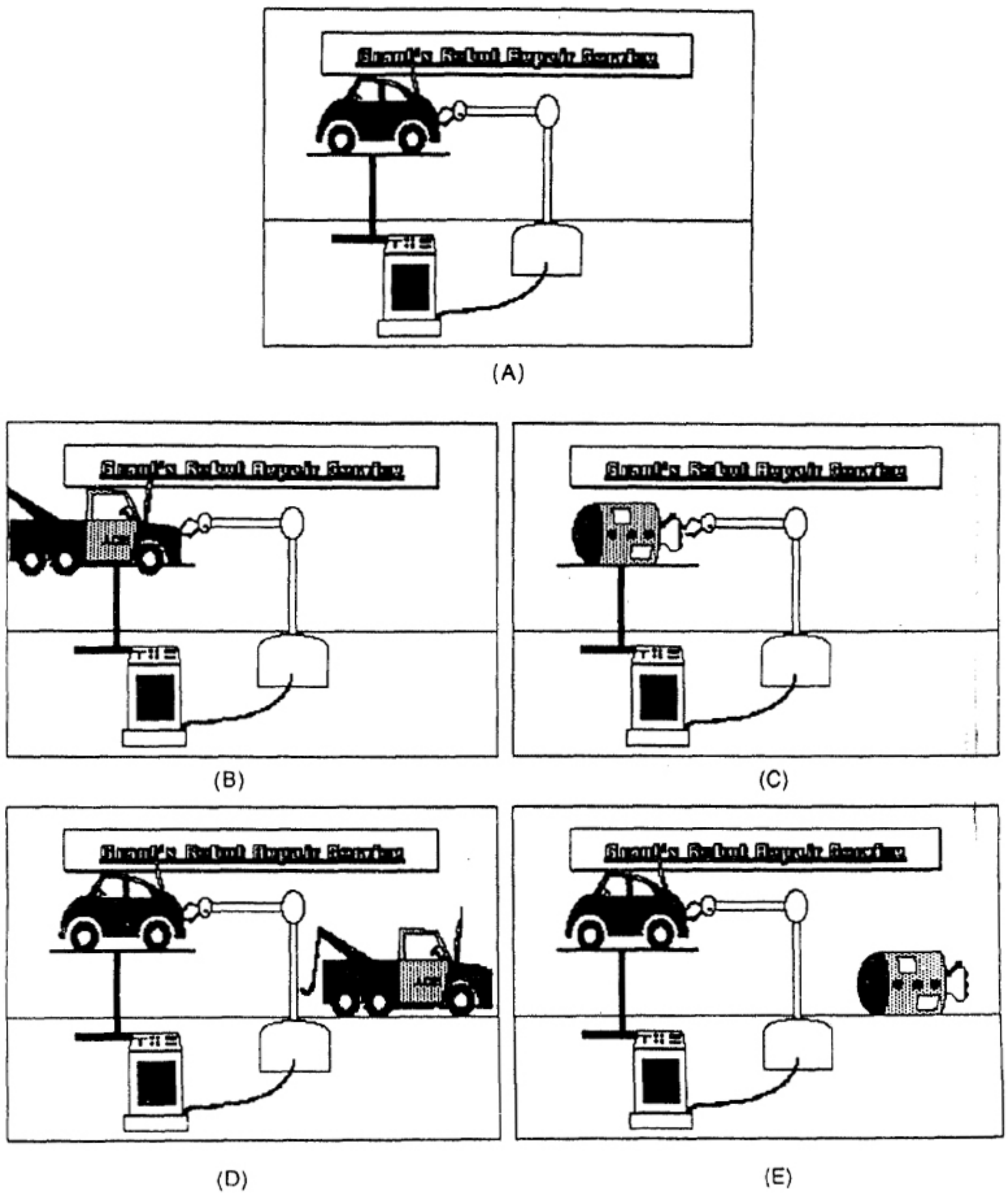

Figure 2.5: Sample set of pictures used by Markman and Gentner (1996).

2.1.4 Nonmonotonicity

Both spatial (Shepard, 1962b; Carroll & Wish, 1974) and feature-set (Tversky, 1977) accounts of similarity assume that similarity is a monotonically increasing function of the number of features shared by compared items. According to these accounts, adding matching features to two items should never decrease their similarity.

An experiment conducted by Goldstone (1996) demonstrates nonmonotonicities that suggest that adding matching features to two items can decrease their similarity if the matching features promote correspondences that are inconsistent with correspondences between similar objects. Section 2.2.5 discusses in detail situations called cross-mappings, in which two correspondences are inconsistent (Gentner & Toupin, 1986; Markman & Gentner, 1993a).

Subjects rated the similarity between two pairs of butterflies using a scale from one (low similarity) to nine (high similarity). The pairs were designed as described in Section 2.1.2 (see Figure 2.3), except the butterflies' bodies varied in color instead of shading, and body color was the only dimension that varied systematically (all other dimensions were randomized). Two nonmonotonicities arose when a butterfly was more similar overall to a butterfly with a different body color than a butterfly with the same body color.

The first nonmonotonicity was between methods XY→XB and XY→XX (see Figure 2.3). Both methods change the body color of one of the butterflies: XY→XB changes color Y to color B, which is a new body color, and XY→XX changes color Y to color X, which matches the body color of the less similar butterfly in the other pair. Although method XY→XX results in one more shared feature than method XY→XB, this shared feature promotes a correspondence between dissimilar butterflies that is inconsistent with a correspondence between similar butterflies. The mean similarity ratings for methods XY→XB and XY→XX were 6.57 and 6.41, respectively; similarity decreased with the addition of a matching feature. However, this result is not conclusive evidence for nonmonotonicity because it can be accounted for if features such as "have identical body colors" are permitted (Tversky, 1977). There is evidence that such relational features are used by people when making similarity judgments (Gentner & Markman, 1995; Goldstone et al., 1991). This also accounts for previous findings where subjects judge "XX" to be more similar to "YY" than to "XY" (Goldstone et al., 1989; Medin et al., 1990; Medin, Goldstone, & Gentner, 1993).

The second nonmonotonicity was between methods XY→AB and XY→YB (see Figure 2.3). Method XY→AB results in no matching body colors, whereas method XY→YB results in one body color that matches, but the matching body color belongs to dissimilar butterflies and is inconsistent with a correspondence between similar butterflies. The mean similarity ratings for methods XY→AB and XY→YB were 5.59 and 5.44, respectively. Again, similarity decreased with the addition of a matching feature, but unlike the apparent nonmonotonicity between methods XY→XB and XY→XX, this nonmonotonicity is not accounted for by subjects' use of relational features.

2.1.5 Asymmetry

People's judgments of similarity often depend on the direction of the comparison. For example, people rate the similarity of North Korea to Red China as higher than the similarity of Red China to North Korea (Tversky, 1977; Tversky & Gati, 1978). Such asymmetries in similarity judgments occur in domains ranging from music perception (Bartlett & Dowling, 1988) to self-other comparisons (Catrambone, Beike, & Niedenthal, 1996; Holyoak & Gordon, 1983).

According to Tversky's (1977) focusing hypothesis, asymmetries in similarity judgments occur because the target of a directional comparison is the focus of attention. As a result, distinctive features of the target count more against similarity than distinctive features of the base. In comparisons where one item has a larger or more salient set of distinctive features than the other item, similarity is lower when the former item is in the target position than when it is in the base position. For example, because more distinctive information is known about Red China than North Korea, Red China (the target) is less similar to North Korea (the base) than North Korea (the target) is similar to Red China (the base).

Like the focusing hypothesis, Ortony's (1979) salience imbalance model derives asymmetries from differential salience of features of the target and the base. Whereas the focusing hypothesis derives asymmetries from distinctive features, the salience imbalance model proposes that asymmetry results from the salience of common features being higher in the base than in the target (Ortony, 1979; Ortony, Vondruska, Foss, & Jones, 1985). For example, time is more similar to a river than a river is similar to time because the common features of time and a river (e.g., they both flow) are more salient for rivers than for time.

Traditional spatial approaches to similarity (Shepard, 1962b, 1974; Carroll & Wish, 1974), which define the similarity between two items as inversely related to the distance between them in psychological space, are inconsistent with asymmetries in similarity judgments because distance is a symmetric relation (Tversky, 1977). However, asymmetry can be derived from a symmetric relation plus a differential bias associated with the compared items (Holman, 1979; Krumhansl, 1978; Nosofsky, 1991). For example, North Korea is more similar to Red China than Red China is to North Korea because Red China is associated with a larger bias than North Korea. Potential stimulus biases include item density in the surrounding space, frequency of stimulus instantiation, and item prototypicality (Nosofsky, 1991). For instance, Polk, Behensky, Gonzalez, and Smith (2002) manipulated the exposure frequency of colors and found that rarely encountered colors are judged to be more similar to frequently encountered colors than vice versa.

According to reference point models, asymmetries occur when one of the items is a more natural reference point or landmark than the other item (Rosch, 1975; Shen, 1989; Gleitman, Gleitman, Miller, & Ostrin, 1997). Deviant items are perceived as more similar to reference items than vice versa because deviant items are more easy to assimilate.

Reference-point models invoke grammatical constraints that place the deviant item in the figure position of a sentence, and the reference item in the ground position (Talmy, 1978).

People expect an utterance to be informative (Grice, 1975). Comparisons often suggest inferences (Reed, Ernst, & Banerji, 1974; Gick & Holyoak, 1980; Ross, 1987; Novick, 1988; Markman, 1997). The coherence imbalance hypothesis (Gentner & Bowdle, 1994; Bowdle & Gentner, 1997) provides a functional explanation of asymmetry that posits that asymmetries reflect differences in the informativeness of each direction of comparison. Given information precedes new information in an utterance (Clark & Haviland, 1977). It follows that in directional comparisons, new information is projected from the base to the target. The coherence imbalance hypothesis derives asymmetries from differences between the coherence of the target and the base. The coherence imbalance hypothesis predicts that similarity is judged to be greater when the base is more coherent than the target. For example, North Korea is more similar to Red China than vice versa because our knowledge of Red China is more coherent than our knowledge of North Korea. The direction of comparison where North Korea is the target and Red China is the base is more informative than the reverse direction because it enables us to make inferences about the less coherent item (North Korea) based on the more coherent item (Red China). The coherence imbalance hypothesis operationalizes coherence as systematicity, which is the degree to which a concept is structured by a system of predicates governed by higher-order causal or explanatory relations (Gentner, 1983; Kintsch & van Dijk, 1978; Trabasso & van den Broek, 1985; Keil, 1989; Murphy & Medin, 1985). Systematicity is discussed in detail in Section 2.2.4.

2.1.6 The Time Course of Similarity

The similarity between two items varies during the time course of the comparison process (Goldstone & Medin, 1994; Goldstone, 1996). To test whether judgment time influences similarity judgments, Goldstone and Medin (1994) manipulated judgment time by giving subjects different deadlines for responding whether two pairs of butterflies were the same or different. Similarity was assumed to increase as a function of the percentage of trials subjects incorrectly responded that two pairs were the same.

Figure 2.6 shows two sample comparisons from the experiment. In Figure 2.6 (a), the top butterfly in the pair on the left shares four MIPs with the corresponding bottom butterfly in the pair on the right, and the bottom butterfly in the pair on the left shares three MIPs with the corresponding top butterfly in the pair on the right. In Figure 2.6 (b), the top butterfly in the pair on the left shares three MIPs with the corresponding bottom butterfly in the pair on the right, and the bottom butterfly in the pair on the left shares three MIPs with the corresponding top butterfly in the pair on the right and one MOP (body shading) with the non-corresponding butterfly in the pair on the right. In both comparisons, seven features are shared by the two pairs; in Figure 2.6 (a) all seven of the shared features are MIPs whereas in Figure 2.6 (b) six of the shared features are MIPs and one of the shared features is a MOP. The central finding is that when subjects are required to respond quickly to meet a one second deadline, similarity is equally influenced by MIPs and MOPs, but when subjects are given longer deadlines of 1.84 and 2.68 seconds, the relative influence of MIPs over MOPs increases with time. Comparisons with more globally consistent feature matches (e.g., Figure 2.6 (a)) become increasingly similar with time compared to comparisons with inconsistent local feature matches (e.g., Figure 2.6 (b)). Thus, early in processing all matching features are equally salient, but over time global constraints on correspondences come into play and accentuate the importance of MIPs over MOPs.

Goldstone (1996) demonstrated nonmonotonicities that suggest that adding matching features to two items can decrease their similarity if the matching features promote correspondences that are inconsistent with correspondences between similar objects (see Section 2.1.4). A subsequent experiment demonstrates that these nonmonotonicities are modulated by judgment time. The experiment used the six trial types described in Sec-

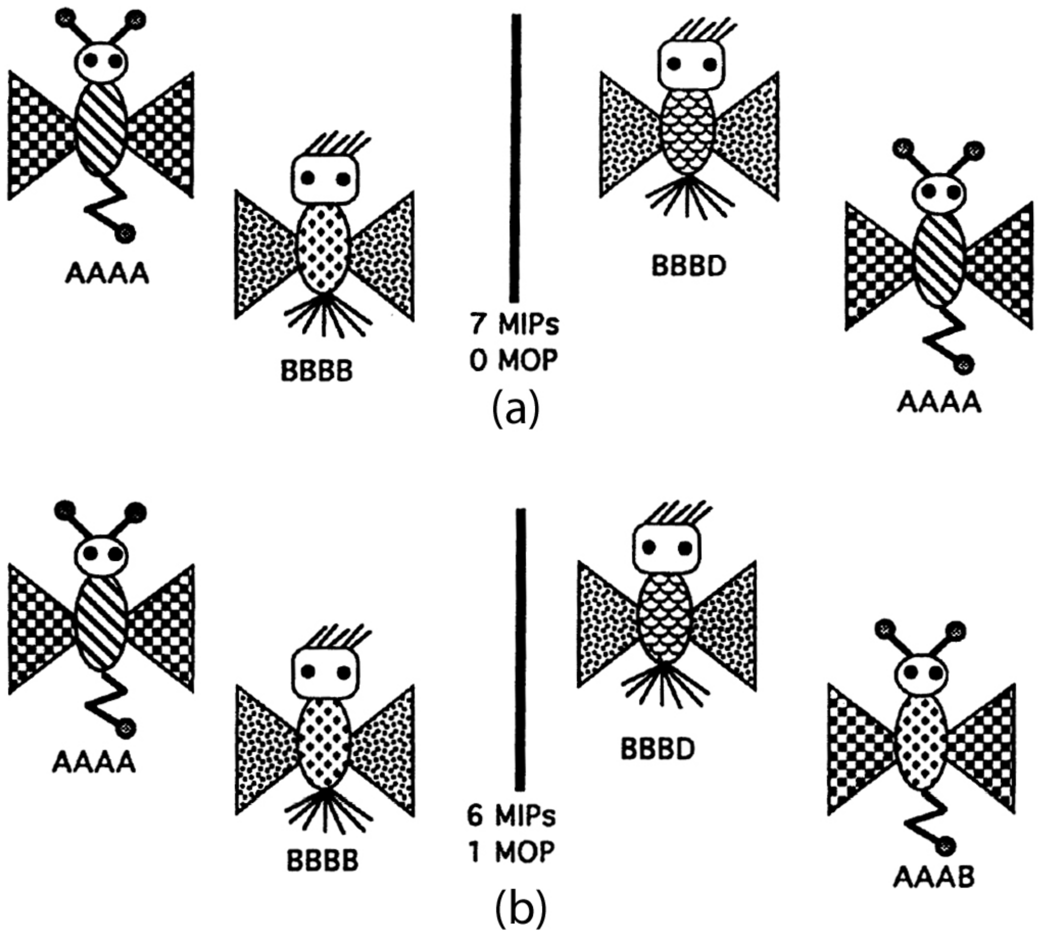

Figure 2.6: Sample comparisons from Goldstone and Medin (1994).

tion 2.1.2 (see Figure 2.3), except the butterflies' bodies varied in color instead of shading, and body color was the only dimension that varied systematically (all other dimensions were randomized). Processing time was manipulated by presenting trials for different durations. Each trial, two pairs of butterflies were simultaneously presented and remained on the screen for 1.5, 3, or 5 seconds, after which the screen was erased. Subjects then rated the similarity between the two pairs of butterflies using a scale from one (low similarity) to nine (high similarity).

Subjects' similarity judgments exhibited nonmonotonicities for the intermediate duration, but not for the short or long durations. A significant nonmonotonicity was found between methods XY→XB and XY→XX for intermediate durations. A nonsignificant nonmonotonic trend was also found between methods XY→AB and XY→YB for intermediate durations. This trend was significant in a replication with simpler materials and a wider range of display durations (Goldstone, 1996). Both the difference between methods XY→XB and XY→XX and the difference between methods XY→AB and XY→YB were in the opposite direction for the short and long durations, but the difference was significant only for the short duration.

These results are consistent with Goldstone and Medin's (1994) finding that early in processing all matching features are equally salient, but in time global constraints on object correspondences come into play and accentuate MIPs over MOPs. For the short duration, MOPs increase similarity as much as MIPs and similarity is determined by the total number of matching features. Thus, an additional MOP increases similarity for the short duration. For the long duration, global constraints on correspondences come into play and accentuate MIPs over MOPs. Thus, an additional MOP does not influence similarity for the long duration. For the intermediate duration, MOPs compete strongly with MIPs for attention because proper correspondences are not fully established and can be weakened by conflicting correspondences. While an additional MOP has some positive influence on similarity for the intermediate duration, it has a stronger negative influence on similarity by drawing attention away from MIPs. Thus, an additional MOP has the net effect of decreasing similarity only for the intermediate duration.

Subjects' similarity judgments exhibited two additional patterns. First, similarity ratings for the long duration were significantly higher than for the short and intermediate durations. This is consistent with evidence that subjects are more likely to revise earlier similarity ratings by increasing rather than decreasing them (Medin et al., 1993). Secondly, the general result from Goldstone and Medin (1994) was replicated; MIPs relative to MOPs were more influential as judgment time increased. The six methods of altering the butterflies can be divided into two groups: methods XY→XX, XY→YB, and XY→YX result in at least one MOP whereas the other three methods result in only MIPs. The mean similarity ratings for the former group for the short, intermediate, and long durations were 6.14, 6.00, and 6.10, respectively. For the latter group, the mean ratings were 6.45, 6.63, and 6.69. These ratings exhibit a significant interaction between display duration and the influence of MOPs versus MIPs.

2.2 Analogical Mapping

At the core of analogy is mapping of mental representations. Mapping is the process of determining which elements of one representation correspond to which elements of another representation. If one representation has m elements and the other representation has n elements, then there are 2^{mn} potential mappings between the two representations. For example, if each representation has 5 elements, then there are over 10 million potential mappings. This problem is similar to that of stereoscopic vision, which requires determining correspondences between images from each eye (Marr & Poggio, 1976). Marr and Poggio propose several constraints on the visual system that collectively lead to specific mappings between images. Likewise, psychological constraints on analogical mapping lead to specific mappings between mental representations and have received considerable attention in the literature. The theoretical framework for much of this research is Gentner's

(1983) structure-mapping theory of analogy. The remainder of Section 2.2 describes constraints on analogical mapping and discusses empirical findings and benchmark phenomena related to these constraints.

2.2.1 Relational Similarity

Similarity comparisons are rooted in semantic commonalities between compared items. Analogy differs from other kinds of similarity in the type of commonalities that are shared by compared items (Gentner, 1983; Collins & Burstein, 1989). According to Gentner and Clement (1988):

> The basic intuition is that an analogy is a mapping of knowledge from one domain (the base) into another (the target), which conveys that a system of relations that holds among the base objects also holds among the target objects. Thus, an analogy is a way of noticing relational commonalities independently of the objects in which those relations are embedded. According to this view, in interpreting an analogy people seek a common relational structure. (pp. 312-313)

Analogical mappings are based on relations rather than independent object descriptions common to compared analogs. Corresponding objects need not resemble each other, but rather are placed in correspondence because they play the same roles in relations common to both analogs. For example, Rutherford's analogy between the atom and the solar system is based on relational structure shared by both systems (Gentner, 1983; Falkenhainer et al., 1989). The nucleus is bigger than the electrons as the Sun is bigger than the planets. The nucleus attracts the electrons causing the electrons to revolve around the nucleus as the Sun attracts the planets causing the planets to revolve around the Sun. Correspondences between the nucleus and the Sun and between the electrons and the planets are based on the objects' roles in this shared relational structure. Very few attributes are shared by the electrons and the planets or the nucleus and the Sun.

A taxonomy of comparisons can be designated according to whether similarity is based on common relational structure, object descriptions, or both (Gentner, 1983, 1989). Analogies are based on common relational structure rather than object descriptions. Mere-appearance comparisons are based on object descriptions rather than relational structure. Literal similarity comparisons are based on both relational structure and object descriptions. Figure 2.7 shows the similarity space formed by varying the degree to which compared items share relations versus object descriptions (Gentner & Clement, 1988).

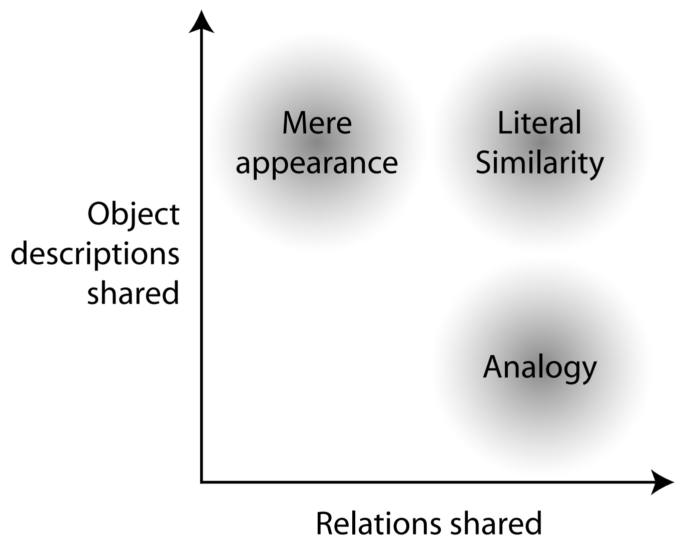

Figure 2.7: Similarity space showing different kinds of similarity in terms of degree of relations versus object descriptions shared by compared items.

To test the primacy of relations in analogy, Gentner and Clement (1988) presented subjects with the comparisons listed in Table 2.2. The materials represent the three kinds of comparisons described above: analogies, mere-appearance comparisons, and literal similarity comparisons. In the analogies, relations are shared by compared items. For example, a camera is like a tape-recorder because a camera captures images as a tape-recorder captures sound. In the mere-appearance comparisons, features are shared by compared items. For example, the Sun is like an orange because both are round and of similar color. In the

Table 2.2: Materials used by Gentner and Clement (1988) to explore relational similarity.

Analogy:	The moon is like a lightbulb.
	A camera is like a tape-recorder.
	A ladder is like a hill.
	A cloud is like a sponge.
	A roof is like a hat.
	Treebark is like skin.
	A tire is like a shoe.
	A window is like an eye.
Mere-appearance:	Jellybeans are like balloons.
	A cloud is like a marshmallow.
	A football is like an egg.
	The sun is like an orange.
	A snake is like a hose.
	Soap suds are like whipped cream.
	Pancakes are like nickels.
	A tiger is like a zebra.
Literal similarity:	A doctor is like a repairman.
	A kite is like a bird.
	The sky is like the ocean.
	A hummingbird is like a helicopter.
	Plant stems are like drinking straws.
	A lake is like a mirror.
	Grass is like hair.
	Stars are like diamonds.

literal similarity comparisons, both relations and features are shared by compared items. For example, a doctor is like a repairman because a doctor fixes injuries as a repairman fixes appliances and a doctor and a repairman are both human.

In Gentner and Clement's experiment, subjects first wrote descriptions of each of the individual items involved in the comparisons. Next, subjects were given the comparisons and were asked to write the meaning of each comparison and rate its aptness, which concerned how clever, interesting, and worthwhile the comparison was. Finally, judges rated the degree to which the item descriptions and comparison interpretations contained relations versus features.

The results suggest that people focus on relations when making comparisons. First, the interpretations of the analogies and literal similarity comparisons contained more relations than features. This is especially noteworthy for the literal similarity comparisons, which could support both relational and feature-based interpretations. Secondly, highlighting of relations over features occurred specifically in the comparison interpretations; the associated item descriptions were high in features as well as relations. Thirdly, subjects' aptness ratings were positively correlated with the degree to which their comparison interpretations contained relations and negatively correlated with the degree to which their comparison interpretations contained features. Lastly, subjects rated the analogies and literal similarity comparisons as more apt than the mere-appearance comparisons. Thus, when making comparisons, people appear to seek relations that are shared by compared items, and the more of these relations they are able to find, the more apt they find the comparison.

Gentner's (1988) relational shift hypothesis proposes that the ability to find relations shared by compared items develops. Before the relational shift at about 6 years of age, children can understand comparisons based on shared features, but after the relational shift, they can understand and prefer comparisons based on shared relations. To test this hypothesis, Gentner repeated Gentner and Clement's experiment described above, but with subjects consisting of three age groups: children aged 5-6 and 9-10 and adults. The findings from the original study were replicated within the adult group. In addition, there was an increase with age in the degree to which comparison interpretations contained relations, but there was no increase with age in the degree to which comparison interpretations contained features. The 5-6-year-olds also did not show the same tendency as the 9-10-year-olds and adults to produce relational rather than feature-based interpretations of the literal similarity comparisons. Unlike the adults, the 5-6-year-olds and the 9-10-year-olds did not

Table 2.3: Sample comparisons and interpretations used by Gentner (1988). R and F denote relational and feature-based interpretations, respectively.

Analogy:	A cloud is like a sponge...
	R) Both can hold water
	F) Both are fluffy
	A tire is like a shoe...
	R) Both cover the bottom of something
	F) Both are made of rubber
Mere-appearance:	Jelly beans are like balloons...
	R) Both are fun at parties
	F) Both are round
	A snake is like a hose...
	R) Both can curl up
	F) Both are wiggley
Literal similarity:	Plant stems are like drinking straws...
	R) Both can be used to get water
	F) Both are thin
	Grass is like hair...
	R) Both cover and protect something
	F) Both are long

display in their aptness ratings a preference for analogies and literal similarity comparisons over mere-appearance comparisons, and there was no correlation between children's aptness ratings and the degree to which their comparison interpretations contained relations or features.

While these results seem to demonstrate a developmental increase in relational focus when making comparisons, the production and rating tasks used may not be equally suited for adults and children. To address these methodological concerns, a subsequent experiment was conducted using a choice task rather than a production task and using a new method for obtaining subjects' ratings. Subjects consisted of three age groups: children aged 4-5 and 7-8 and adults. For each comparison, subjects were given a relational interpretation and a feature-based interpretation and were asked to choose the interpretation they preferred. Table 2.3 gives examples of the comparisons and interpretations used in the experiment. After a choice was made, adults were asked to rate both interpretations on a scale from one (boring) to five (very interesting), and children were asked to indicate their ratings on a vertical "goodness meter."

The results of the experiment support the findings from the previous experiment. In both the choice and rating tasks, there was a developmental shift toward preferring relational interpretations over feature-based interpretations of the analogies and literal similarity comparisons. In contrast, all three age groups preferred the feature-based interpretations of the mere-appearance comparisons, indicating that the developmental shift is specifically relational. Not all kinds of comparisons have the same developmental trajectory. While young children behave like adults with respect to mere-appearance comparisons, a relational focus in both production and comprehension of interpretations of analogies and literal similarity comparisons appears to develop with age.

In the studies discussed above, a preference for relational similarity was demonstrated by subjects' ratings of how apt, boring versus interesting, or generally "good" different kinds of comparisons are. An experiment conducted by Gentner, Rattermann, and Forbus (1993) addresses a candidate functional determinant of these ratings: inferential soundness. Subjects were given pairs of stories and rated their inferential soundness and similarity. Subjects were instructed that inferential soundness concerned how well inferences true of one story would apply to the other story. Each pair of stories consisted of a base story and a story that shared either relations or object descriptions with the base. For example, the base story and relational match in Table 2.4 share relations (e.g., the squirrel is disappointed with the mockingbird as Sam's mother is not at all pleased with him), whereas the base story and the feature-based match in Table 2.4 share features

Table 2.4: Sample stories used by Gentner, Rattermann, and Forbus (1993) to demonstrate a preference for relational similarity.

Base story:	Percy the mockingbird spent the whole warm season chirping and twittering. When it began to get colder Percy visited a squirrel and sang a song for her, expecting to get some of the squirrel's sunflower seeds in return. However, the squirrel was very disappointed in him. "You are a terrible singer!" she yelled. "I'm not giving you any of my wheat." A tear rolled down Percy's cheek, and he vowed to give up singing for good.
Relational match:	Sam travelled all over the world buying beautiful things. When he ran out of money he paid a visit to his mother. However, she was not at all pleased with him. "While I have been hard at work you have been wasting your time," she said. Sam gave her a gift he bought in Tibet, hoping she would give him a loan in return. But she was still not pleased. "I will not give you any of my hard-earned money!" she exclaimed.
Feature-based match:	One unusually warm spell in February Sam the magpie thought "This is my chance." He stood up on the edge of his nest and trilled proudly. His song was so loud and cheerful that it woke a nearby chipmunk. The chipmunk asked for another song. He was so moved by Sam's talents that he forgot it was still winter and decided to go looking for nuts to store.

(e.g., the mockingbird and the magpie are both birds). Consistent with previous findings, comparisons between stories that shared relations were considered more inferentially sound than comparisons between stories that shared features. In addition, subjective similarity was higher between stories that shared relations than between stories that shared features. Thus, the degree to which compared items share relations strongly influences their similarity and the soundness of the comparison.

2.2.2 One-to-One Correspondence

Often in comparisons, an element in one representation might plausibly correspond with more than one element in the other representation. The structural constraint of one-to-one correspondence requires that each element in one representation be placed into correspondence with at most one element in the other representation. For example, when comparing "Tarzan loves Jane" and "Joanie loves Chachi," the constraint of one-to-one correspondence allows for Tarzan to be placed into correspondence with Joanie (because Tarzan and Joanie fill the "lover" roles in their respective "loves" relations) or Chachi (because Tarzan and Chachi are male), but not both Joanie and Chachi.

There is some empirical evidence that people's mappings violate the constraint of one-to-one correspondence. In a study conducted by Spellman and Holyoak (1992), nine percent of subjects who were asked to determine correspondences between the 1991 Persian Gulf War and World War II placed Kuwait (which was invaded by Iraqi in the 1991 Persian Gulf War) into correspondence with two or more of Austria, Czechoslovakia, and Poland (which were invaded by Germany in World War II).

Subjects in studies conducted by Spellman and Holyoak (1996) frequently reported mappings that violated the constraint of one-to-one correspondence. For example, in one study, subjects read descriptions of two fictional planets and generated mappings between them. A schematic diagram of the descriptions of the planets is shown in Figure 2.8. One planet had three countries: Afflu was strong economically and gave economic aid to

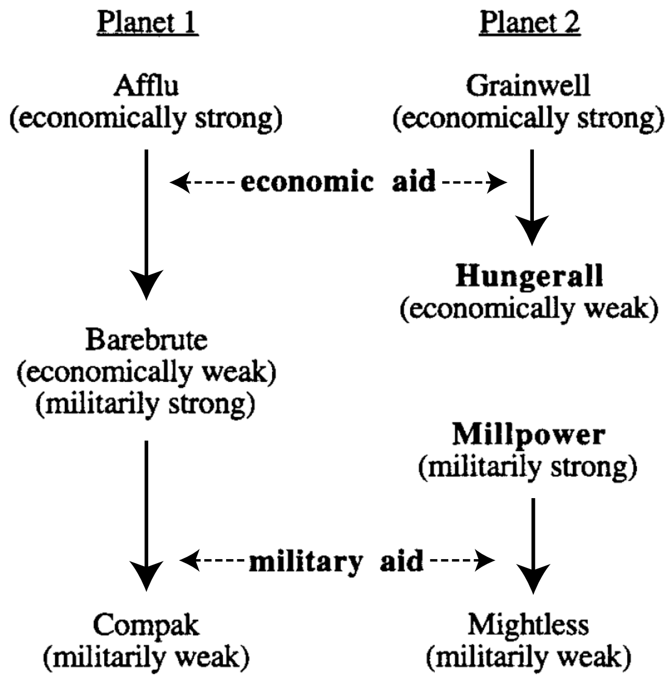

Figure 2.8: Schematic diagram of descriptions used by Spellman and Holyoak (1996). Vertical arrows represent economic or military aid relations.

Barebrute, Barebrute was weak economically but strong militarily and gave military aid to Compak, and Compak was weak militarily. The other planet had four countries: Grainwell was strong economically and gave economic aid to the economically weak Hungerall, and Millpower was strong militarily and gave military aid to the militarily weak Mightless. Thus, Afflu and Compak corresponded with Grainwell and Mightless, respectively, but Barebrute could correspond with Hungerall (because both received economic aid) and Millpower (because both gave military aid). Fifty percent of the subjects violated the constraint of one-to-one correspondence and mapped both Hungerall and Millpower to Barebrute.

A study conducted by Markman (1997) examines the impact of violations of one-to-one correspondence on analogical inference. Subjects played the role of a college dean who is moving from an old school to a new school and were given descriptions of departments in the old school and the new school. A schematic diagram of the descriptions of the departments is shown in Figure 2.9. Each department in the old school was described by two causal statements in which an antecedent leads to some consequent (e.g., excellent teaching in the English department causes oversubscribed classes). Each department in the new school was described by two statements that contained only causal antecedents from the old school (e.g., the Music department has excellent teaching). The potential for violating one-to-one correspondence was afforded by using one causal antecedent from each description of a department in the old school to compose the descriptions of the departments in the new school. For example, both the Music department in the new school and the English department in the old school have excellent teaching, while both the Music department in the new school and the Biology department in the old school have a small number of faculty. Thus, any department in the new school could plausibly correspond with any department in the old school. Consistent with the results described above, the mean number of departments in the old school reported by subjects as corresponding to a department in the new school was 1.41, whereas one-to-one correspondence allows for only one department in the old school to correspond to a department in the new school.

After the correspondence task, subjects were asked to make predictions about what might happen at the new school given what they knew about the new school and the old school. Analogical mapping permits a target representation (e.g., the new school) to be extended by its similarity to a base representation (e.g., the old school) via a process of copying with substitution and generation (Holyoak, Novick, & Melz, 1994). Markman (1997) describes this process as follows:

> Copying with substitution and generation involves taking any element in the base domain for which there is a correspondence and carrying over to the target all representational structure attached to that element. Whenever a correspondence between base and target exists for an element being inferred, that correspondence is substituted into the information being inferred. Relations in

Old School	New School
English department	**Computer Sci. Department**
CAUSE (obtain Eng_faculty grants) (hire Eng_faculty rsch_assts)	(obtain CS_faculty grants)
CAUSE (excellent Eng_faculty teach) (oversubscribed classes)	(infighting CS_faculty)
Biology department	**Music Department**
CAUSE (infighting Bio_faculty) (avoid Bio_faculty offices)	(excellent Mus_faculty teach)
CAUSE (small-number Bio_faculty) (not (perform Bio_faculty advising))	(small-number Mus_faculty)

Figure 2.9: Schematic diagram of descriptions of academic departments used by Markman (1997). Statements are represented using a simplified propositional notation.

> the base that are not in the target are carried over identically. Finally, new target entities can be posited when their existence is required to complete a structure from the base. (p. 376)

For example, consider just the first statements describing the Computer Science department in the new school and the English department in the old school (see Figure 2.9). The following correspondences can be drawn from the target to the base: "obtain"→"obtain," "CS_faculty"→"Eng_faculty," and "grants"→"grants." Copying with substitution and generation begins by copying "CAUSE" (which is attached to "obtain") and "hire" (which is attached to "Eng_faculty") from the base to the target. Because "CS_faculty" in the target corresponds to "Eng_faculty" in the base, "CS_faculty" is substituted for "Eng_faculty" in propositions being copied from the base to the target. Finally, a new entity "Rsch_assts" is posited in the target to complete the propositions copied from the base. Thus, it is inferred that obtaining grants causes the Computer Science faculty to hire research assistants.

Many-to-one mappings where more than one element in the target corresponds to the same element in the base pose a problem for copying with substitution and generation. For example, in Figure 2.9, the Computer Science faculty and the English faculty obtain grants and the Music faculty and the English faculty are excellent at teaching. Suppose both the Computer Science faculty and the Music faculty are placed into correspondence with the English faculty. As described above, the proposition that obtaining grants causes the faculty to hire research assistants is copied from the base to the target. However, a problem arises in substituting for the English faculty. Because both the Computer Science faculty and the Music faculty correspond to the English faculty, it is possible to get inconsistent substitutions resulting in nonsensical inferences such as the Computer Science faculty obtaining grants causes the Music faculty to hire research assistants.

Subjects' inferences revealed no such inconsistent substitutions, suggesting that subjects did not use many-to-one mappings for copying with substitution and generation. While at first glance the results of the correspondence and inference tasks may seem contradictory, they are not. In the correspondence task, subjects reported one-to-many mappings where an element in the target corresponds to more than one element in the base, not many-to-one mappings where more than one element in the target corresponds to the same element in the base. Unlike many-to-one mappings, one-to-many mappings provide a single element in the target to substitute for several elements in the base, which results in consistent substitutions. Thus, it appears that one-to-one correspondence is a soft constraint on mapping (see Section 2.1.6 for supporting evidence with respect to similarity judgments), but a strict one-to-one-or-more constraint is applied prior to copying with substitution and generation.

2.2.3 Parallel Connectivity

The structural constraint of parallel connectivity requires that the arguments of corresponding predicates themselves be placed into correspondence. For example, consider the comparison between "Tarzan loves Jane" and "Joanie loves Chachi." These two propositions can be represented in predicate notation as LOVES(TARZAN, JANE) and LOVES(JOANIE, CHACHI). According to the constraint of relational similarity (see Section 2.2.1), the LOVES relation in one representation is placed into correspondence with the LOVES relation in the other representation. Based on this correspondence, the constraint of parallel connectivity requires that TARZAN and JOANIE correspond because both are the first argument to their respective LOVES relation, and that JANE and CHACHI correspond because both are the second argument to their respective LOVES relation. In other words, the two people who love are placed into correspondence and the two people who are loved are placed into correspondence. Importantly, parallel connectivity allows nonidentical and even dissimilar representational elements to be placed in correspondence if they fill similar roles in matching relational structure. For instance, the corresponding people in the current example have opposite genders.

Parallel connectivity is so fundamental to analogical mapping that virtually every study in the literature implicitly provides evidence supporting its validity. For example, each study described in Section 2.2.2 suggests that parallel connectivity underlies analogical mapping. In Spellman and Holyoak's (1992) study, subjects placed Iraq and Kuwait into correspondence with Germany and Austria, respectively, because Iraq invaded Kuwait in the 1991 Persian Gulf War as Germany invaded Austria in World War II. In Spellman and Holyoak's (1996) study, subjects placed fictional countries into correspondence based on the roles they played in either giving or receiving economic or military aid. In Markman's (1997) study, subjects placed faculty in the new school into correspondence with faculty in the old school if they played like roles in shared causal antecedents.

Perhaps parallel connectivity is most apparent in proportional analogies common in tests such as the Scholastic Aptitude Test, Graduate Record Examination, and Miller Analogies Test. The task is to complete an analogy of the form "A is to B as C is to *blank*" such that the relation between C and *blank* is the same as the relation between A and B and parallel connectivity is preserved. For example, "lungs" is an appropriate completion of "fish are to gills as humans are to *blank*" because fish breath with gills as humans breath with lungs. Via parallel connectivity, the shared "breathes with" relation provides for correspondences between fish and humans and between gills and lungs. It has been argued that the ability to perceive relations that support parallel connectivity underlies human creativity and discovery (Hofstadter, 1995; Indurkhya, 1992; Gentner et al., 1997).

2.2.4 Systematicity

Higher-order relations, which take other relations rather than entities as arguments, can encode important relationships such as implications or causal relationships. For example, in the study conducted by Markman (1997) described in Section 2.2.2, the higher-order relation CAUSES in the proposition CAUSES(OBTAINS(ENGLISH FACULTY, GRANTS), HIRES(ENGLISH FACULTY, RESEARCH ASSISTANTS)) encodes the causal relationship between obtaining grants and hiring research assistants. Causal information can be helpful in integrating information about new domains (Murphy & Allopenna, 1994). Importantly, mental representations involving several layers of nested higher-order relations can contain highly interconnected relational structures.

The studies described so far suggest that analogical mappings are based on relations common to compared items. However, not all relations are equally influential in analogical mapping. The structural constraint of systematicity states that mappings that preserve large systems of interconnected relations are preferred over mappings that place small, disjointed systems of relations into correspondence. Thus, among common relations, people place into correspondence those that are part of a coherent system of relations interconnected by higher-order relations. For example, in Rutherford's analogy between the atom and the solar system, the mass difference between the nucleus and the electrons corresponds with the mass difference between the Sun and the planets because these relations are part of systematic relational structure representing that differences in mass together with mutual attraction cause the smaller entities to revolve around the larger entity. The higher-order causal relation plays a critical role in connecting this relational structure. In contrast, the temperature difference between the Sun and the planets is irrelevant to the analogy because there is no mappable systematic relational structure associated with the difference (Falkenhainer et al., 1989).

There is substantial evidence that systematicity aids analogical transfer. In a study of analogical problem solving, Holyoak and Koh (1987) found that subjects were more successful at transferring a solution from one problem to an analogous problem when the problems involved similar causal relationships. Gentner and Schumacher (1986) and Schumacher and Gentner (1988) found that systematic structure facilitates transfer from a device model to an analogous device. Gentner and Toupin (1986) found that 9-year-olds were more successful at transferring a story plot between two sets of characters when the story included explicit causal structure.

Table 2.5: Stories from Gentner et al. (1993) showing the influence of systematicity on judgments of inferential soundness and similarity.

Base story:	Percy the mockingbird spent the whole warm season chirping and twittering. When it began to get colder Percy visited a squirrel and sang a song for her, expecting to get some of the squirrel's sunflower seeds in return. However, the squirrel was very disappointed in him. "You are a terrible singer!" she yelled. "I'm not giving you any of my wheat." A tear rolled down Percy's cheek, and he vowed to give up singing for good.
First-order match:	Sam travelled all over the world buying beautiful things. When he ran out of money he paid a visit to his mother. However, she was not at all pleased with him. "While I have been hard at work you have been wasting your time," she said. Sam gave her a gift he bought in Tibet, hoping she would give him a loan in return. But she was still not pleased. "I will not give you any of my hard-earned money!" she exclaimed.
Systematic match:	Sam travelled all over the world buying beautiful things. When he ran out of money he paid a visit to his mother and gave her a gift he bought while in Tibet, hoping she would give him a loan in return. However, his mother was not at all pleased. "You don't deserve any money of mine!" she exclaimed. "This is a piece of junk!"

Systematicity also influences the evaluation of analogies (Gentner & Landers, 1985; Rattermann & Gentner, 1987; Gentner et al., 1993). In an experiment conducted by Gentner et al. (1993), subjects were given pairs of stories and rated their inferential soundness and similarity. Subjects were instructed that inferential soundness concerned how well inferences true of one story would apply to the other story. Each pair of stories consisted of a base story and a story that varied in the systematicity of the relational structure it shared with the base. For example, the base story and first-order match in Table 2.5 share only first-order relations (e.g., the squirrel is disappointed with the mockingbird as Sam's mother is not at all pleased with him), whereas the base story and the systematic match in Table 2.5 share first-order relations interconnected by higher-order relations (e.g., the mockingbird sings a terrible song causing the squirrel to be disappointed with him as Sam gives his mother a piece of junk causing her to be not at all pleased with him). Comparisons between stories that shared systematic relational structure were considered more inferentially sound than comparisons between stories that shared only first-order relations. In addition, subjective similarity was higher between stories that shared systematic relational structure than between stories that shared only first-order relations. Thus, the systematicity of the mapping between compared items influences their similarity and the soundness of the comparison.

The set of different relational matches between compared items can be large. Systematicity constrains which relational matches people include in their analogical mappings (Gentner & Clement, 1988; Clement & Gentner, 1991). Gentner and Clement (1988) point out that systematicity or a variant of it has been posited as a selection filter by several researchers (Burstein, 1983; Hofstadter, 1984; Indurkhya, 1985; Kedar-Cabelli, 1985; Van Lehn & Brown, 1980; Winston, 1980, 1982). To demonstrate that mapping choices are guided by systematicity, Clement and Gentner (1991) gave subjects analogous stories that shared two key first-order relations and asked them choose which of the key relations contributed most to the analogy. The key relations differed in whether they were part of

Base: ***The Toms***	**Target**: ***The Robots***	
	Version 1	**Version 2**
Consume minerals with underbellies	**Gather data with probes**	**Gather data with probes**
Exhaust minerals in one spot and must relocate on the rock	**Exhaust data in one place and must relocate on the planet**	Internal computers over-heat when gather a lot of data
So stops using underbelly	*So stops using probes*	*So stops using probes*
Born with inefficient underbelly	Designed with delicate probes	Designed with inefficient probes
Underbelly adapts and becomes specialized for one rock	Robots cannot pack probes to survive flight to a new planet	**Probes adapt and become specialized for one planet**
So underbelly can't function on new rock	*So probes can't function on new planet*	*So probes can't function on new planet*

Note. Key facts are shown in italics. Matching causal information is shown in boldface.

Table 2.6: Relational structure of stories used by Clement and Gentner (1991) to demonstrate that mapping choices are guided by systematicity.

a shared systematic relational structure governed by a higher-order causal relation. For example, the base and each version of the target in Table 2.6 share the two key relations shown in italics. In Version 1 of the target, the shared relation “stops using” is a causal consequent of the shared relational structure shown in bold, but the shared relation “can’t function” is an isolated match. In Version 2 of the target, the shared relation “can’t function” is a causal consequent of the shared relational structure shown in bold, but the shared relation “stops using” is an isolated match. On 79 percent of the trials, subjects chose the key relation that was causally connected to a shared system of relations over the isolated key relation (reported by Markman & Gentner, 2000). In contrast, when subjects read only the target stories and were asked to choose the most important key relation, they did not show the same tendency. This suggests that the importance of a relation shared by analogous items depends on the degree to which it is interconnected with other matching relations. That is, systematicity acts as a selection constraint on analogical mapping.

2.2.5 Flexibility

Markman and Gentner (2000) set out three benchmark phenomena that characterize different aspects of the flexibility of analogical mapping: interactive interpretation, multiple interpretation, and cross-mapping.

The benchmark of interactive interpretation states that different comparisons may highlight different aspects of the items being compared. For example, the roundness of the moon is highlighted when it is compared to a ball, whereas the brightness of the moon is highlighted when it is compared to a lamp (James, 1985). In the study conducted by Clement and Gentner (1991) described in Section 2.2.4, the interpretation of the base story (i.e., which key relations were important) depended on which target story was compared to the base. Studies conducted by Goldstone (1994) and Markman and Gentner (1997) demonstrate that aspects of compared items are highlighted in comparisons that cast them as MIPs rather than MOPs (see Section 2.1.2) or alignable differences rather than nonalignable differences (see Section 2.1.3).

The benchmark of multiple interpretation states that the same comparison can be interpreted in different ways. For example, in the study conducted by Gentner (1988) described in Section 2.2.1, older children and adults given a comparison (e.g., a tire is like a shoe) were able to appreciate both the feature-based interpretation (e.g., both are made of rubber) and the relational interpretation (e.g., both cover the bottom of something), but preferred the relational interpretation.

The benchmark of cross-mapping is a special case of the benchmark of multiple interpretation. Cross-mapping occurs when objects with similar features play different roles in shared relational structure (Gentner & Toupin, 1986). For example, Markman and Gentner (1993b) and Markman (1996) gave subjects two pictures in which cars played different relational roles. In one picture, a car was towing a boat, while in the other picture, a truck was towing a car. Before indicating which object in one picture (i.e., the car or the boat) corresponded to the car in the other picture, subjects rated either the artistic merit of the pictures or the similarity or difference of the pictures. While subjects who rated the artistic merit of the pictures tended to place the two cars into correspondence because they looked similar, subjects who compared the pictures tended to place the car into correspondence with the boat because both were being towed. This suggests that people attend to relational structure when making comparisons. People can perceive both feature-based and relational interpretations of cross-mappings, but comparisons highlight the latter.

2.2.6 Scale

People are able to process analogies involving large representations. For example, in studies conducted by Gentner and Landers (1985) and Gentner et al. (1993), subjects judged the soundness of an analogy between the two stories shown in Table 2.7. Each story involves several entities (e.g., Karla, hunter, bow, arrows, feathers, deer) and many interconnected

Table 2.7: Stories from Gentner and Landers (1985) and Gentner et al. (1993) illustrating the ability to process analogies involving large representations.

Base story:	Karla, an old hawk, lived at the top of a tall oak tree. One afternoon, she saw a hunter on the ground with a bow and some crude arrows that had no feathers. The hunter took aim and shot at the hawk but missed. Karla knew the hunter wanted her feathers so she glided down to the hunter and offered to give him a few. The hunter was so grateful that he pledged never to shoot at a hawk again. He went off and shot deer instead.
Target story:	Once there was a small country called Zerdia that learned to make the world's smartest computer. One day Zerdia was attacked by its warlike neighbor, Gagrach. But the missiles were badly aimed and the attack failed. The Zerdian government realized that Gagrach wanted Zerdian computers so it offered to sell some of its computers to the country. The government of Gagrach was very pleased. It promised never to attack Zerdia again.

relations (e.g., Karla giving the hunter her feathers causes the hunter to be grateful, which causes the hunter to promise not to attack Karla). Detailed representations of these stories are given in Section 5.3.12.

Chapter 3

Models of Similarity and Analogy

3.1 Models of both Similarity and Analogy

3.1.1 The Structure-Mapping Engine

The Structure-Mapping Engine (SME; Falkenhainer et al., 1989) was designed as a simulation of Gentner's (1983) structure-mapping theory. SME takes as input two propositional representations composed of entities, predicates, and functions. Entities are objects and constants (e.g., SUN). Predicates can represent attributes or relations. An attribute is a unary predicate that describes some property of an entity (e.g., HOT(SUN)). A relation is a predicate with multiple arguments that can be entities or other predicates (e.g., CAUSE(GRAVITY, ATTRACTS)). Whereas predicates map into truth values, functions map one or more entities into another entity, and can be used to represent dimensions (e.g., MASS(SUN)). Falkenhainer et al. (1989) refer to both predicates and functions as functors.

Figure 3.1 shows representations used by Falkenhainer et al. (1989) to simulate Rutherford's analogy between the atom and the solar system. SME determines correspondences between the two representations given as input using a local-to-global alignment process. SME begins by finding all possible local correspondences between the two representations. Match hypotheses are created between identical functors. For example, SME

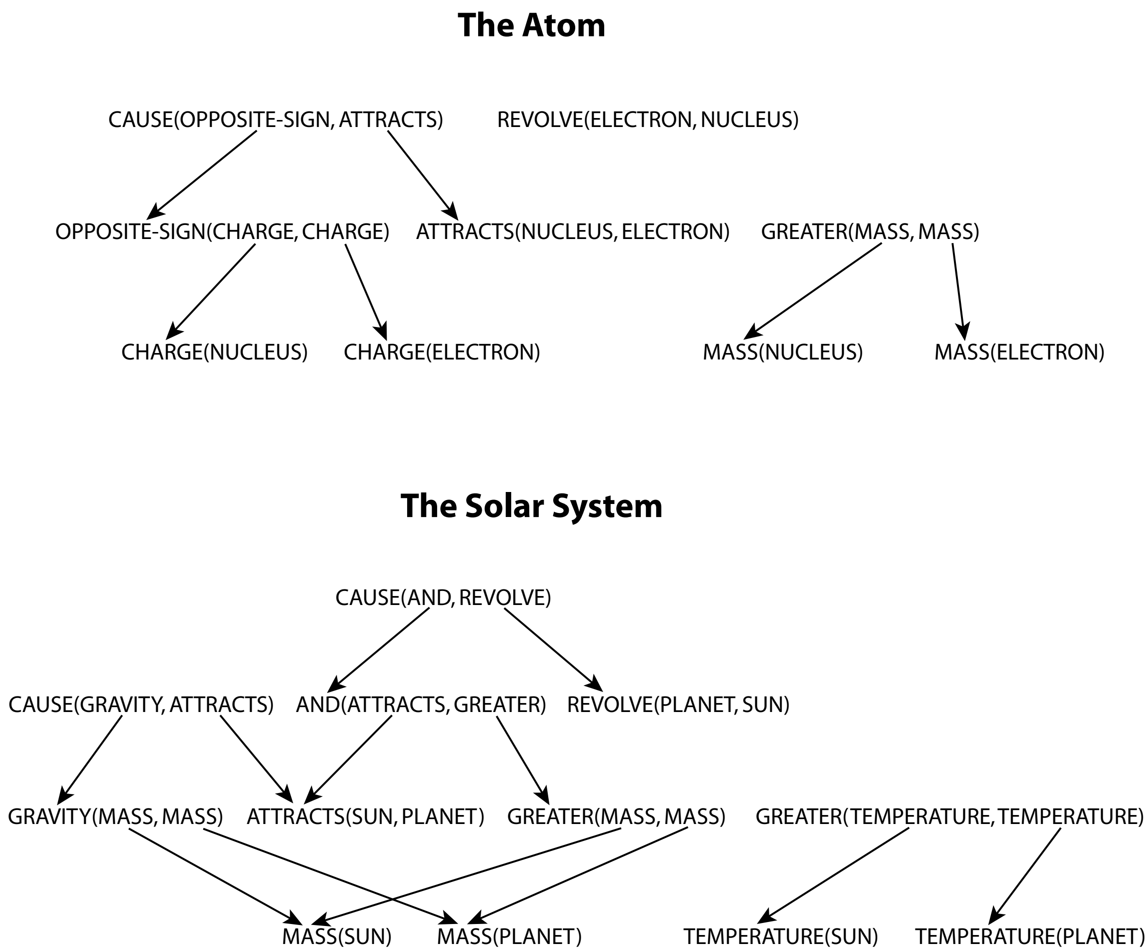

Figure 3.1: SME's representations for Rutherford's analogy between the atom and the solar system.

creates the following match hypotheses between the atom and the solar system: CAUSE matches with each CAUSE, REVOLVE matches REVOLVE, ATTRACTS matches ATTRACTS, GREATER matches each GREATER, and each MASS matches each MASS. Consistent with the structural constraint of parallel connectivity (see Section 2.2.3), SME also creates match hypotheses between entities or functions that are corresponding arguments of matching functors (e.g., MASS matches TEMPERATURE because they are corresponding arguments of matching GREATER relations). The process of using corresponding functors to place their arguments into correspondence is applied recursively, ending with match hypotheses between entities (e.g., NUCLEUS matches SUN).

At this stage, the mapping typically violates the structural constraint of one-to-one correspondence by containing inconsistent match hypotheses that place an element in one representation into correspondence with more than one element in the other representation. For example, NUCLEUS matches SUN based on matching ATTRACTS relations, but also matches PLANET based on matching MASS functions. SME produces one or a few globally consistent mappings by coalescing combinations of consistent local correspondences. This can be done exhaustively or using a greedy merge algorithm (Forbus & Oblinger, 1990). Importantly, all final mappings generated by SME strictly impose the structural constraints of parallel connectivity and one-to-one correspondence.

SME calculates structural evaluation scores for each mapping using a cascade-like algorithm that favors systematic correspondences between identical representational elements. The algorithm has three steps. First, initial scores are assigned to each individual match hypothesis. A match hypothesis is assigned a positive initial score if it places identical functors into correspondence, and a score of 0 otherwise. Next, in a top-down fashion, a percentage of the score for match hypotheses between functors is added to the score for match hypotheses between their corresponding arguments. This results in higher scores for match hypotheses at the bottom of deep, interconnected relational structures. Finally, the structural evaluation score for each mapping is the sum of the scores of the match hypotheses which comprise its correspondences.

SME selects the mapping with the highest structural evaluation score as the preferred mapping. For example, SME generates three different mappings for Rutherford's analogy (Falkenhainer et al., 1989). The mapping with the highest structural evaluation score places the nucleus into correspondence with the Sun and the electrons into correspondence with the planets based on the mass difference in the atom playing the same role as the mass difference in the solar system. This mapping supports the inference that differences in mass together with mutual attraction cause the electrons to revolve around the nucleus. However, because SME does not place nonidentical relations into correspondence, the mapping does not recognize that electrical force (i.e., OPPOSITE-SIGN) corresponds with gravity. The mapping with the second highest structural evaluation score has the same entity correspondences, but places the difference in mass into correspondence with the difference in temperature. The structural evaluation score for this mapping is lower because no corresponding relational structure is connected to the temperature difference, and because mass and temperature are different functions. The mapping with the lowest structural evaluation score places the mass of the electron into correspondence with the mass of the sun and the mass of the nucleus into correspondence with the mass of the planet. This mapping receives the lowest structural evaluation score because the entity correspondences do not permit any relational correspondences.

One limitation of SME is its insensitivity to the influence of MOPs in judgments of similarity. For example, whereas people rate the similarity between Figure 3.2 (a) and Figure 3.2 (b) as higher than the similarity between Figure 3.2 (a) and Figure 3.2 (c), SME generates the same structural evaluation score for both comparisons. This is because SME strictly adheres to the structural constraint of one-to-one correspondence. In both comparisons, SME's preferred mapping places OBJECT1 into correspondence with OBJECT1 and OBJECT2 into correspondence with OBJECT2 based on the relation BESIDE and the attributes GREY and BLACK. In the comparison between Figure 3.2 (a) and Figure 3.2 (b),

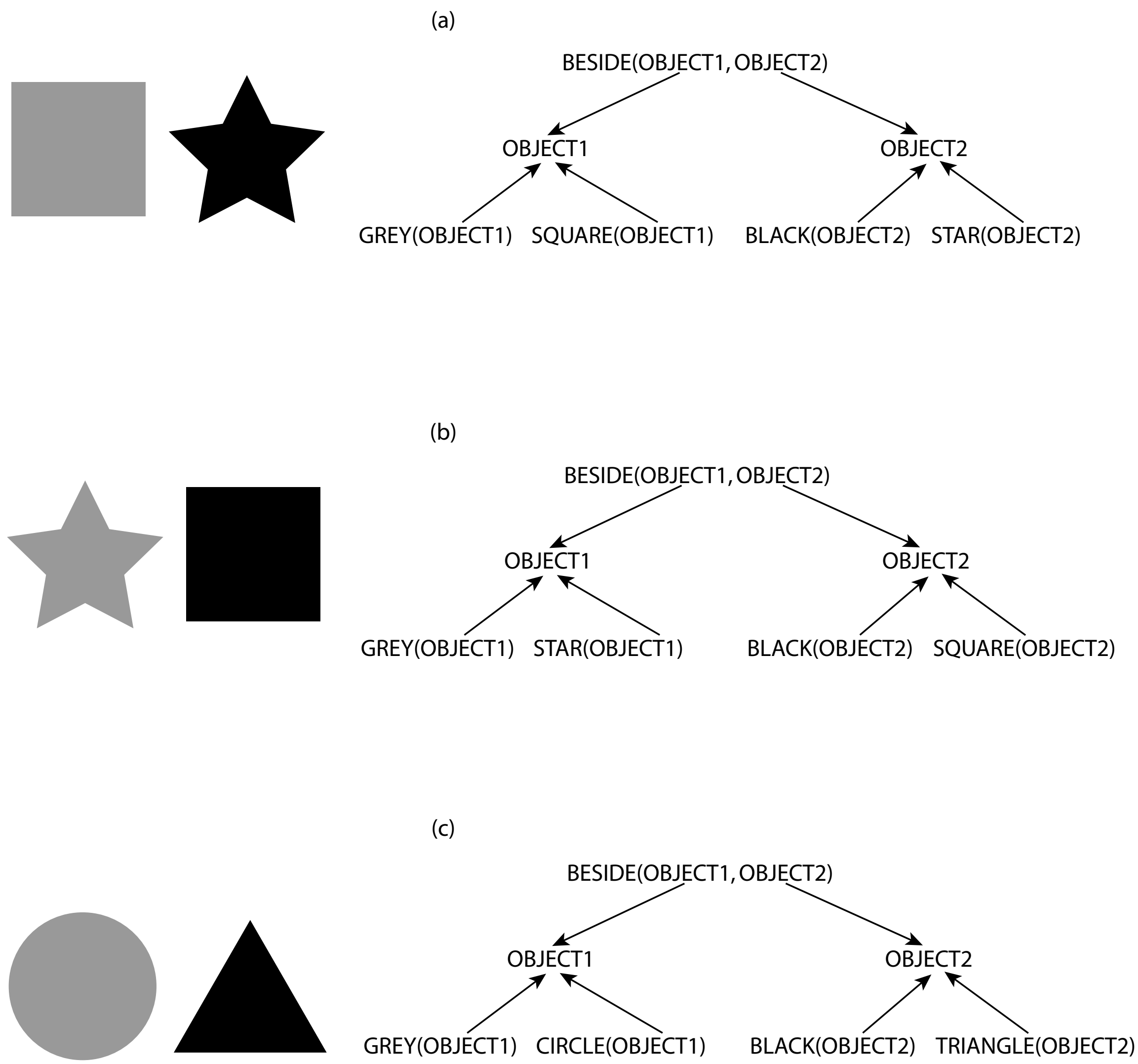

Figure 3.2: Geometric configurations and representations illustrating SME's insensitivity to MOPs.

the match hypotheses placing SQUARE into correspondence with SQUARE and STAR into correspondence with STAR are excluded from the final mapping because they are inconsistent with the dominant entity correspondences. Thus, unlike people's similarity judgments, SME's structural evaluation score ignores these MOPs and treats both comparisons the same.

A more general limitation of SME as a model of similarity and analogy is its focus on product as opposed to process (Love, Rouder, & Wisniewski, 1999). In terms of Marr's (1982) levels of analysis, SME straddles the computational and algorithmic levels. As opposed to generating tentative mappings throughout the time course of the comparison process, SME generates interpretable mappings only after its final phase of computation. As a result, SME has difficulty accounting for response-time data such as those described in Section 2.1.6. In addition, SME does not include capacity constraints. This precludes application of SME to working memory manipulations and related individual or group differences data. Connectionist Analogy Builder (CAB; Larkey & Love, 2003) was developed in part to address these shortcomings, though CAB and SME are motivated by similar ideas about the product of human comparison (e.g., Gentner, 1983, 1989).

3.1.2 Connectionist Analogy Builder

CAB is a process model of comparison that makes time-course predictions as well as predictions concerning the role of working memory in the comparison process. CAB is a connectionist model that is capable of mapping complex relational structures involved in analogical reasoning.

CAB takes as input two directed graphs that can be translated to and from frame systems (Minsky, 1981). For example, Figure 3.3 shows frame and graph representations for the comparison between a grey square beside a black star and a grey star beside a black square. Translating between frames and the graph structure that CAB operates over is accomplished by first taking the frames (in this case, BESIDE, OBJECT1, and OBJECT2) and

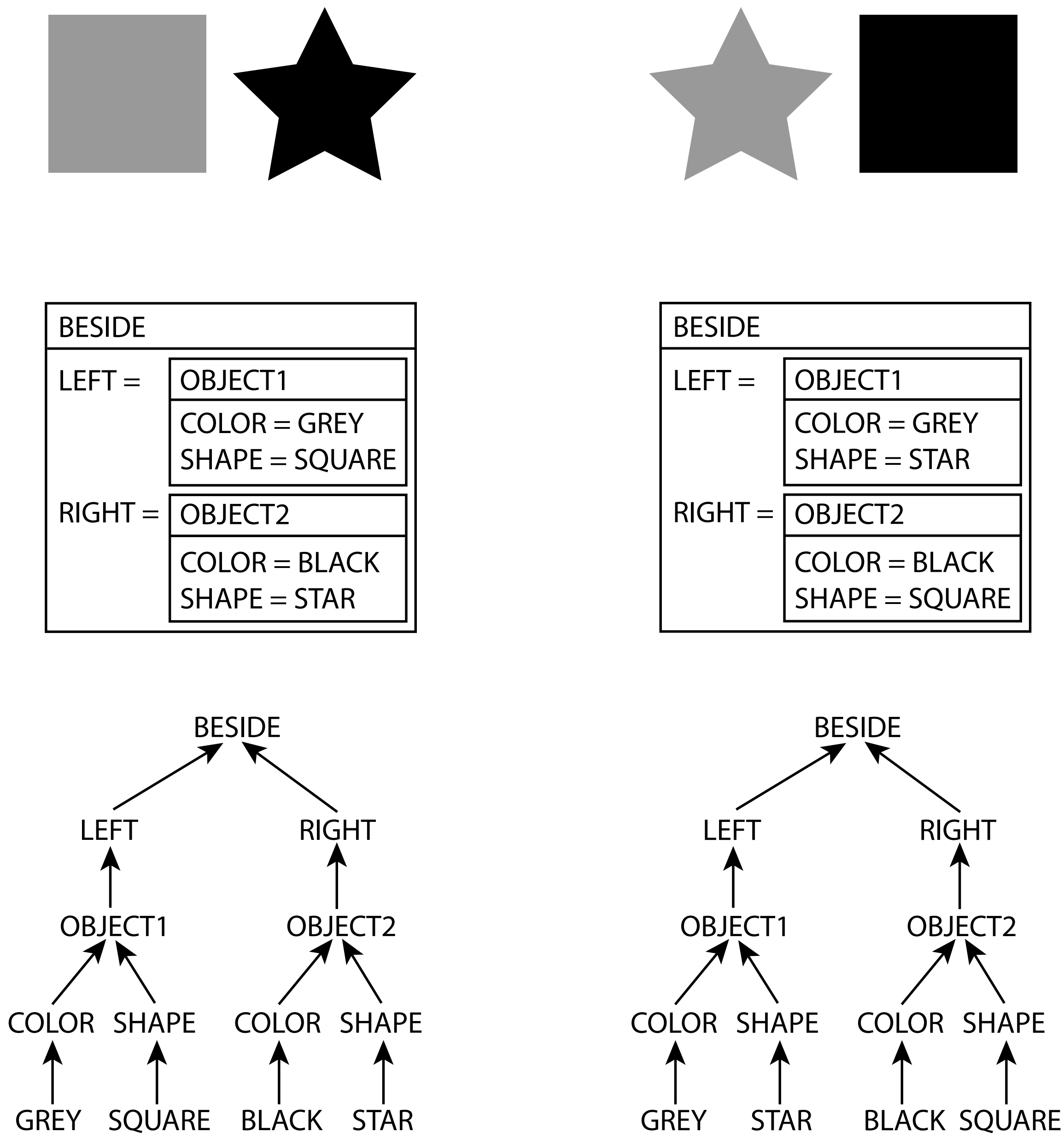

Figure 3.3: Frame and graph representations for the comparison between a grey square beside a black star and a grey star beside a black square.

replacing each with a vertex that denotes the frame. Then, additional vertices are created for each slot of a frame, and edges point from these slot vertices to their associated frame vertices. For instance, LEFT, and RIGHT are slots of the frame BESIDE (see Figure 3.3). Finally, each filler is instantiated as a vertex in the graph, and edges point from these filler vertices to the slots they fill. In the present case, vertices are created for the fillers GREY, SQUARE, BLACK, and STAR, and edges point from each filler vertex to the COLOR or SHAPE vertex denoting the slot it fills. Note that frames can fill slots in other frames. For example, the frame OBJECT1 fills the slot LEFT in the frame BESIDE. Representations involving several layers of nested frames can result in highly systematic graph structures.

The edges in CAB's directed graphs capture the structure of mental representations. For example, the graph on the left of Figure 3.3 represents a grey square beside a black star, while the graph on the right of Figure 3.3 represents a grey star beside a black square. Both graphs contain the same vertices, but in the graph on the left, edges point from SQUARE to SHAPE to OBJECT1 and from STAR to SHAPE to OBJECT2, whereas in the graph on the right, edges point from STAR to SHAPE to OBJECT1 and from SQUARE to SHAPE to OBJECT2. In other words, in the graph on the left, the property of being square shaped is bound to OBJECT1 and the property of being star shaped is bound to OBJECT2, whereas in the graph on the right, the property of being star shaped is bound to OBJECT1 and the property of being square shaped is bound to OBJECT2.

The directions of the edges are important because they specify paths between vertices within a graph. For example, the path from the vertex GREY to the vertex SQUARE in the graph on the left of Figure 3.3 is $(+, +, -, -)$, where $+$ and $-$ denote whether a "hop" in the traversal from GREY to SQUARE is with or against the direction of the edge. The length of this path is four because the traversal involves four edges. Paths encode purely structural relationships between vertices in terms of the edge directions connecting them. Information about the vertices visited along the paths is discarded.

CAB determines mappings via a dynamic process of interactive activation among

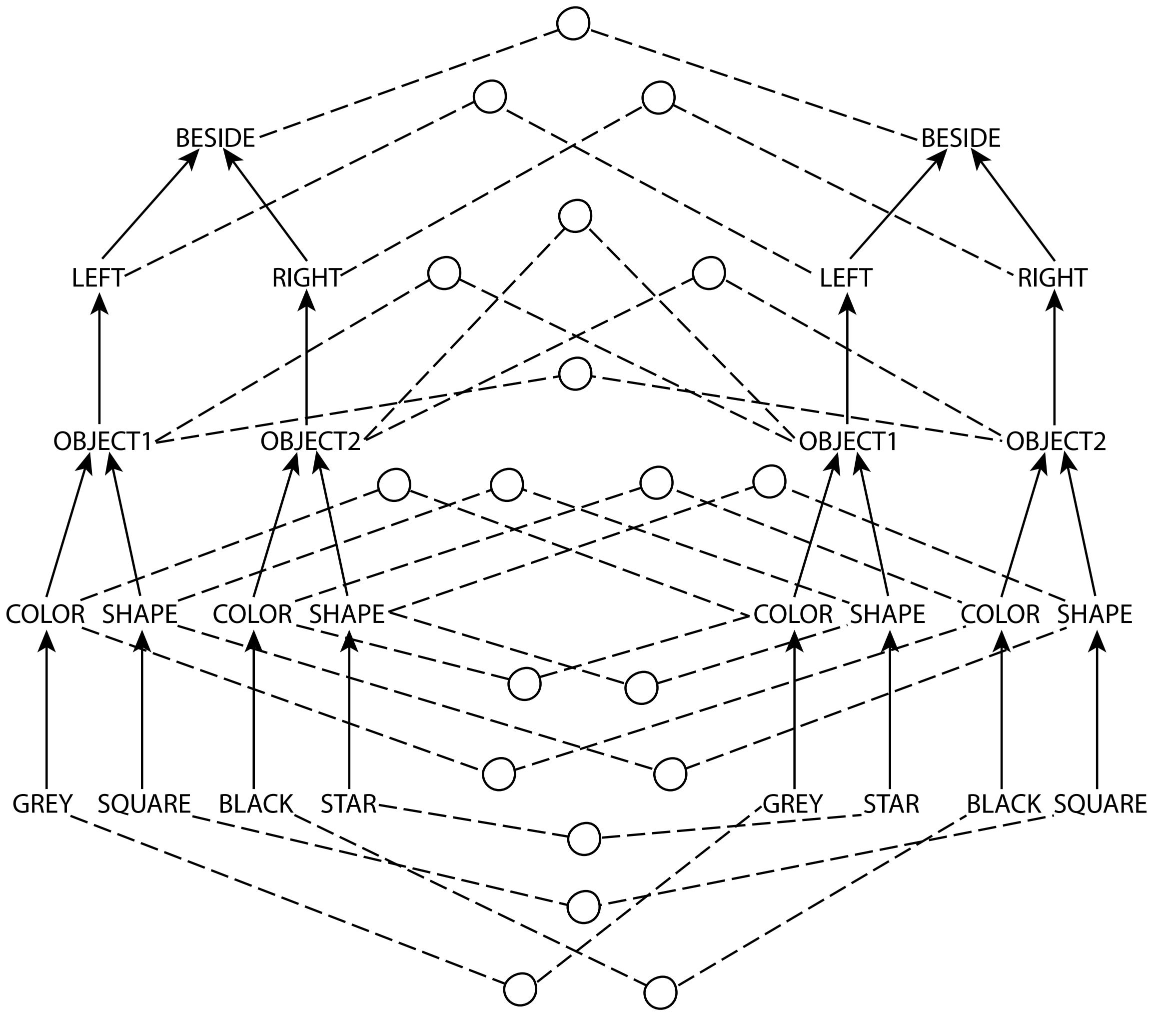

Figure 3.4: Nodes that are initialized with positive activations in the comparison between a grey square beside a black star and a grey star beside a black square. For clarity, connections between nodes are not shown.

correspondences. CAB's architecture can be conceptualized as a network of nodes that represent possible correspondences between the two graphs given as input. The activation of a node indicates the strength of the correspondence it represents. Activation levels range between zero (minimal correspondence) and one (maximal correspondence).

The influence of semantics on mapping is captured by initial node activations. If a node places semantically equivalent vertices into correspondence, then its activation is initialized to a positive model parameter. Otherwise, node activations are initialized to zero. For example, Figure 3.4 shows the nodes that are initialized with positive activations in the comparison between a grey square beside a black star and a grey star beside a black square. From this starting point, activation spreads through the network and CAB learns and unlearns correspondences.

Nodes mutually excite each other if the paths between their associated vertices are the same. For example, in Figure 3.5, the node placing the OBJECT1 vertices into correspondence has an excitatory connection with the node placing the GREY vertices into correspondence because the path from OBJECT1 to GREY in the graph on the left (i.e., $(-,-)$) is the same as the path from OBJECT1 to GREY in the graph on the right. In contrast, the node placing OBJECT1 on the left into correspondence with OBJECT2 on the right has no connection with the node placing the GREY vertices into correspondence because the path from OBJECT1 to GREY on the left (i.e., $(-,-)$) is different than the path from OBJECT2 to GREY in the graph on the right (i.e., $(+,+,-,-,-,-)$). The ability to appreciate correspondences that are related by long paths allows CAB to display a preference for mappings that exhibit systematicity (see Section 2.2.4). Considering correspondences related by short paths establishes parallel connectivity (see Section 2.2.3).

The strength of excitatory connections decreases exponentially with the length of the associated paths. For example, in Figure 3.5, the node placing the OBJECT1 vertices into correspondence also has an excitatory connection with the node placing the BLACK vertices into correspondence, but this connection is weaker than the connection with the

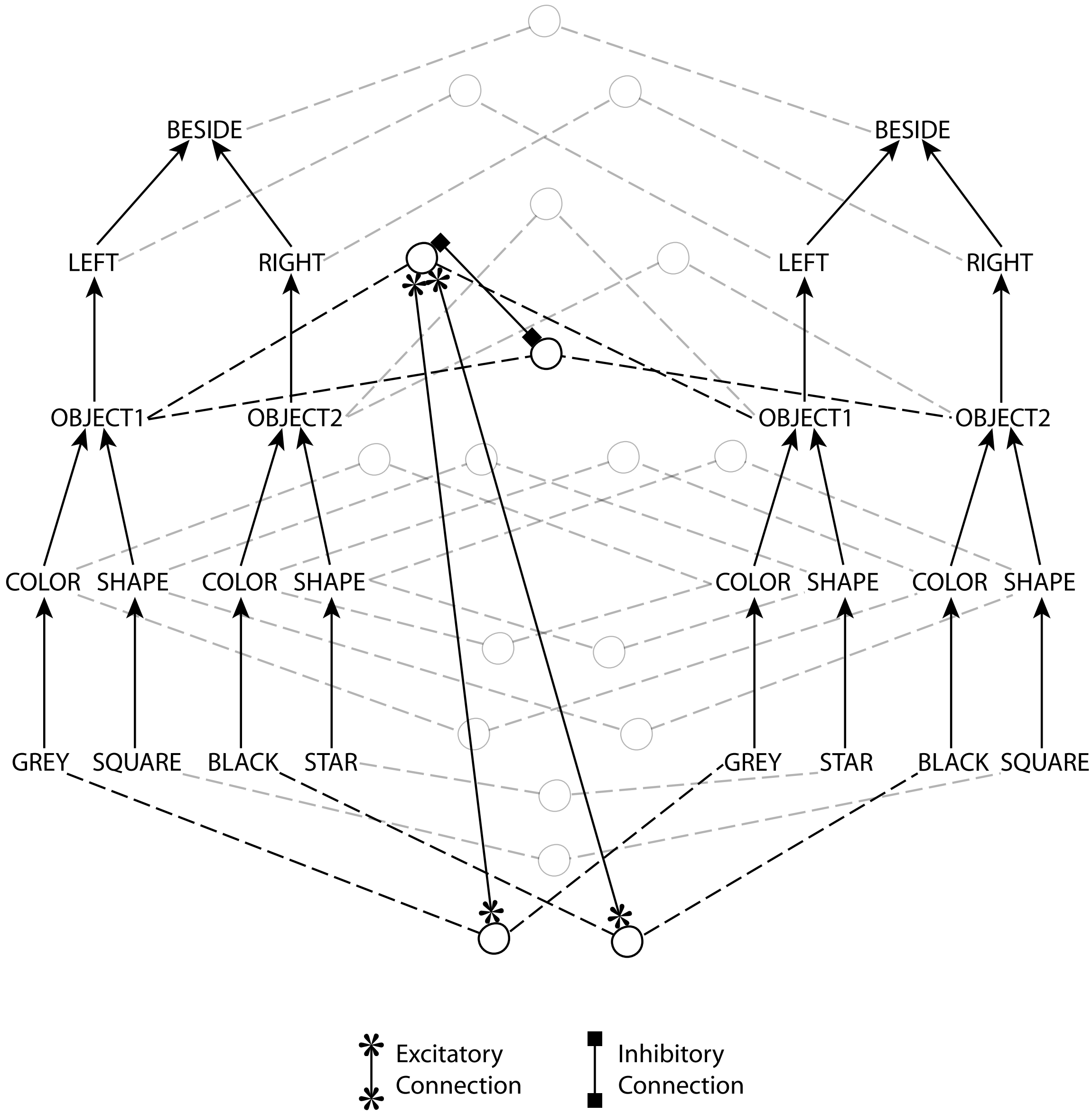

Figure 3.5: Examples of excitatory and inhibitory connections in CAB's network.

node placing the GREY vertices into correspondence because the former connection involves paths of length six (i.e., $(+, +, -, -, -, -)$), whereas the latter connection involves paths of length two (i.e., $(-, -)$). The extent to which excitatory connections weaken with path length is governed by a model parameter. This parameter serves as a proxy for working memory capacity because it governs how much information is considered simultaneously.

Nodes also compete for excitation and are inhibited by other nodes. Excitation placing two vertices into correspondence is suppressed unless it is the maximal excitation received by any node associated with either of the vertices. For example, in Figure 3.5, excitation for the node placing OBJECT1 on the left into correspondence with OBJECT2 on the right is suppressed if the node placing the OBJECT1 vertices into correspondence receives more excitation. This winner-takes-all rule imposes a strict one-to-one correspondence constraint on excitation. In addition, activations compete with each other to establish one-to-one correspondences. For example, in Figure 3.5, the node placing the OBJECT1 vertices into correspondence inhibits the activation of the node placing OBJECT1 on the left into correspondence with OBJECT2 on the right. As a result, CAB shows a strong preference for one-to-one correspondences, though CAB's adherence to the constraint of one-to-one correspondence is not as absolute as SME's.

Because the mapping process is dynamic and incremental, CAB generates mappings and similarity scores throughout the time course of the comparison process. Similarity is calculated using the strength of correspondences between objects to weight the influence of their matching features. For example, in Figure 3.5, the influence of the matching GREY vertices is weighted by the activation of the node placing the OBJECT1 vertices into correspondence. Formally, if m is the total number of objects in one representation and n is the total number of objects in the other representation, then

$$\text{similarity} = \sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij} M_{ij}, \tag{3.1}$$

where A_{ij} is the activation of the node placing object i into correspondence with object j and M_{ij} is the number of matching features shared by object i and object j. Thus, matching features increase similarity to the extent they describe objects placed into correspondence. CAB's ability to account for people's similarity judgments is assessed in Section 5.1.

3.2 Models of Similarity

3.2.1 Multidimensional Scaling and the Contrast Model

Two of the most influential approaches to similarity are spatial approaches, as exemplified by multidimensional scaling (MDS; Shepard, 1962b, 1962a), and feature-set approaches, as exemplified by Tversky's (1977) Contrast Model.

MDS models can take as input any measure of pairwise proximity between items including similarity ratings, dissimilarity ratings, counts of how often items are confused with each other, correlation coefficients, and joint probabilities. The output of MDS models is a dimensionally organized metric space in which items are represented as points.

MDS models define the similarity between compared items as inversely related to the distance between their associated points in space. For example, Shepard (1987) argues that the similarity between compared items is an inverse exponential function of the distance between their associated points in space. The distance between points a and b is typically defined as the p-norm of the vector difference between a and b:

$$d(a, b) = \|a - b\|_p = \left(\sum_{i=1}^{n} |a_i - b_i|^p \right)^{1/p}, \tag{3.2}$$

where n is the dimensionality of the space, a_i is the value of dimension i for a, b_i is the value of dimension i for b, and p determines the specific norm to be used. If $p = 2$, $d(a, b)$ is the length of the line connecting a and b (i.e., the Euclidean distance). If $p = 1$, $d(a, b)$ is the sum of the distances between a and b on each dimension (i.e., the city-block distance). People's judgments of similarity between items involving psychologically nonseparable dimensions (e.g., color brightness and saturation) are often best captured by the Euclidean distance between the items, whereas people's judgments of similarity between items involving psychologically separable dimensions (e.g., shape and color) are often best captured by the city-block distance between the items (Attneave, 1950).

The class of distance metrics defined by Equation 3.2 satisfy the axioms of minimality (i.e., $d(a, b) \geq d(a, a) = 0$), symmetry (i.e., $d(a, b) = d(b, a)$), and the triangle inequality (i.e., $d(a, b) \leq d(a, c) + d(b, c)$). However, people's judgments of similarity systematically violate all three axioms (Tversky, 1977; Gati & Tversky, 1982; Tversky & Gati, 1982).

According to the minimality axiom, the similarity of an item to itself is the same for all items, and this similarity is more than the similarity of any item to a different item. However, pairs of identical objects are more confusable (i.e., more similar) if the objects are complex than if they are simple (Tversky, 1977). Using reaction time for same-different judgments as a measure of similarity, Podgorny and Garner (1979) found that not all letters are equally similar to themselves (e.g., W is less similar to W than S is to S), and that the similarity of a letter to itself can be less than the similarity of a letter to a different letter (e.g., the similarity between W and itself is less than the similarity between O and C). Gilmore, Hersh, Caramazza, and Griffin (1979) found that the letter M is less likely to be recognized as an M than as an H.

According to the symmetry axiom, the similarity between two items is the same regardless of the direction of comparison. However, peoples judgments of similarity often depend on the direction of the comparison. Asymmetries in similarity judgments are described in Section 2.1.5.

According to the triangle inequality, the similarity between two items is greater than or equal to the sum of the similarities between each item and a third item. The triangle inequality can be violated when two items have no features in common, but each of the two items has a different feature in common with a third item (Tversky & Gati, 1982).

For example, a ball and a lamp are not very similar, but a ball and the moon are similar because both are round and a lamp and the moon are similar because both are bright (James, 1985). Thus, the similarity between a ball and a lamp is less than the similarity between a ball and the moon plus the similarity between a lamp and the moon.

As an alternative to spatial approaches to similarity, Tversky (1977) proposed the Contrast Model, which represents items as sets of features. According to the the Contrast Model, similarity increases as a function of common features and decreases as a function of distinctive features of compared items. Formally, let A be the set of features representing a, B be the set of features representing b, $A \cap B$ be the set of features a and b have in common, $A - B$ be the set of features a has that b does not, and $B - A$ be the set of features b has that a does not. Then the Contrast Model defines the similarity of a to b as

$$S(a, b) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A), \tag{3.3}$$

where θ, α, and β are nonnegative model parameters that determine the relative weight of each component of similarity, and f is a nonnegative, monotonically increasing scale that reflects the salience of features and is often assumed to be additive.

The Contrast Model can account for violations of the axioms described above. For example, the Contrast Model accounts for asymmetric similarity because $A - B$ may be different than $B - A$, and as a result, $f(A - B)$ may be different than $f(B - A)$. For instance, let c be North Korea and d be Red China. Then the similarity of North Korea to Red China is

$$S(c, d) = \theta f(C \cap D) - \alpha f(C - D) - \beta f(D - C), \tag{3.4}$$

and the similarity of Red China to North Korea is

$$S(d, c) = \theta f(D \cap C) - \alpha f(D - C) - \beta f(C - D). \tag{3.5}$$

Because set intersection is symmetric, $f(C \cap D) = f(D \cap C)$; however, because Red China has more salient distinctive features than North Korea, $f(C-D) < f(D-C)$. Thus, with $\alpha > \beta$, the Contrast model predicts that the similarity of North Korea to Red China is greater than the similarity of Red China to North Korea (i.e., $S(c,d) > S(d,c)$).

One problem with the Contrast Model is that it predicts that similarity is a monotonically increasing function of the number of features shared by compared items. According to this prediction, adding matching features to two items should never decrease their similarity. However, Goldstone (1996) has demonstrated nonmonotonicities in people's similarity judgments (see Section 2.1.4).

A general limitation shared by MDS models and the Contrast Model is that (like SME) they describe the product of people's similarity judgments more than the process people use to reach these judgments. In terms of Marr's (1982) levels of analysis, MDS models and the Contrast Model are squarely positioned at the computational level. While the abstract and static nature of these models has allowed them to be incorporated into many other cognitive theories (e.g., Nosofsky, 1986 and Osherson, 1990), these qualities ultimately limit their ability to make predictions about the time-course of the comparison process (see Section 2.1.6).

Another limitation shared by spatial and feature-set accounts of similarity is that neither spatial coordinates nor feature sets have a truly general capacity for binding representational elements to other representational elements (Hummel, 2000; Marcus, 1998; Markman, 1999; Hummel & Holyoak, 1997). Capturing such bindings requires structured representations (see Section 2.1.1). One problem associated with this limitation is that because the Contrast Model lacks a mechanism for binding features to particular entities, the influence of a feature shared by multiple entities is ambiguous. For example, when comparing a scene with a red car beside a red fire hydrant to a scene with a red truck beside a red stop sign, it is not clear how many times the $f(A \cap B)$ term in Equation 3.3 should count the common "red" feature. A more severe problem associated with this limitation and shared by MDS models and the Contrast Model is that comparing items involves not simply matching dimension values or features, but determining correspondences between items (see Section 2.1.2 and Section 2.1.3). The influence of the structure of mental representations on correspondences is reflected in the structural constraints of one-to-one correspondence (see Section 2.2.2), parallel connectivity (see Section 2.2.3), and systematicity (see Section 2.2.4). Goldstone's (1994) Similarity as Interactive Activation and Mapping model was developed to address these limitations of MDS models and the Contrast Model, and is described in Section 3.2.3.

3.2.2 Representational Distortion

According to Representational Distortion (RD; Hahn, Chater, & Richardson, 2003), the similarity between a pair of items is inversely related to the number of basic transformations involved in distorting the representation of one item into the representation of the other item. For example, XXXOO is more similar to OOXXX than OXXXO because OOXXX involves reversing XXXOO, whereas OXXXO involves reversing XXXOO plus a left phase shift of OOXXX (Imai, 1977). Post hoc derivations of different sets of transformations for different subjects have yielded high correlations between the number of transformations and similarity ratings (Wiener-Ehrlich, Bart, & Millward, 1980).

While RD can be tested by examining how well the number of transformations predicts perceived similarity, this approach requires a priori rather than post hoc constraints on what counts as a unitary psychological transformation. These constraints remain elusive, but the central claim of RD that a given transformation has a fixed, non-positive influence on similarity can be straightforwardly tested independent of assumptions about the set of basic transformations (Larkey & Markman, 2004; Narvaez & Markman, 2004). Violations of this prediction are described in Section 5.1.

Another difficulty with RD is that it requires a comparison process in order to define the transformations that change one representation into another. For example, distorting a gray square above a black circle into a gray circle beside a black square involves switching

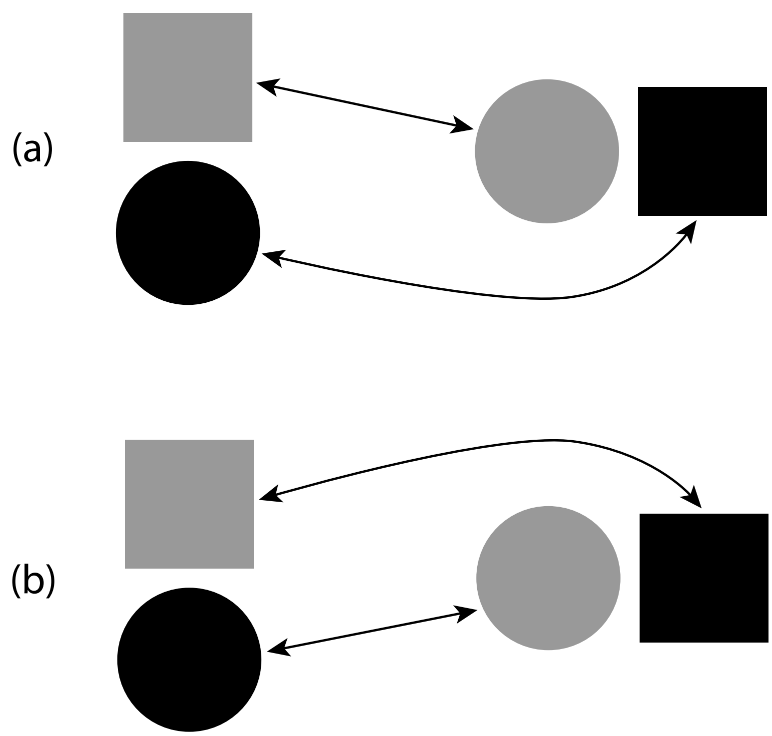

Figure 3.6: Two different interpretations of the same comparison.

shapes if the gray square corresponds to the gray circle and the black circle corresponds to the black square based on color (see Figure 3.6 (a)), but if the gray square corresponds to the black square and the black circle corresponds to the gray circle based on shape (see Figure 3.6 (b)), the transformation involves switching colors. Without principles for creating these correspondences, RD cannot determine which transformations to apply.

3.2.3 Similarity as Interactive Activation and Mapping

Inspired by McClelland and Rumelhart's (1981) interactive activation model of letter perception, Similarity as Interactive Activation and Mapping (SIAM; Goldstone, 1994) is a localist connectionist model that determines similarity via a dynamic process of interactive activation among feature, object, and relational role correspondences. SIAM's architecture consists of a network of nodes that represent all possible feature-to-feature, object-to-object, and role-to-role correspondences between compared stimuli. The activation of a particular node indicates the strength of the correspondence it represents.

Excitatory and inhibitory connections between nodes in the network are set up according to the structure of the stimuli being compared. Objects are placed into correspondence according to correspondences between their features and roles. At the same time, features and roles are placed into correspondence according to correspondences between the objects they describe. There are also excitatory and inhibitory connections between feature-to-feature nodes, between object-to-object nodes, and between role-to-role nodes. These connections are inhibitory if the two nodes taken together place an element in one representation into correspondence with two elements in the other representation, and excitatory otherwise. In addition to being influenced by other nodes, feature-to-feature and role-to-role nodes are influenced by match values that represent perceptually determined similarities between their associated features or roles.

SIAM first creates correspondences between features and roles according to match values. Once features and roles begin to be placed into correspondence, activation spreads through the network and SIAM begins to place objects into correspondence that are consistent with the feature and role correspondences. Once objects begin to be placed into correspondence, activation flows back to feature and role nodes that are consistent with the object correspondences.

SIAM defines similarity as a function of the match values for each feature-to-feature node weighted by the current activation of that node. Matching features increase similarity to a greater extent if their associated feature-to-feature node is highly activated than if it is not. Likewise, mismatching features decrease similarity to a greater extent if their associated feature-to-feature node is highly activated than if it is not. Thus, similarity is a function of common and distinctive features, where the importance of these features is determined by their degree of correspondence.

Whereas SME's adherence to the structural constraint of one-to-one correspondence is strict, SIAM treats one-to-one correspondence as a soft constraint. As a result, and because SIAM allows for different degrees of correspondence (unlike SME's all-or-nothing notion of correspondence), SIAM successfully predicts that both MIPs and MOPs influence similarity, with MIPs having a stronger influence than MOPs (see Section 2.1.2). SIAM also successfully predicts the nonmonotonicities described in Section 2.1.4 (Goldstone, 1996) and the time-course data described in Section 2.1.6 (Goldstone & Medin, 1994; Goldstone, 1996). However, SIAM is not capable of processing comparisons involving systems of relations. Thus, while SIAM can account for many aspects of similarity judgments, it cannot account for analogical mapping.

3.3 Models of Analogy

3.3.1 Analogical Constraint Mapping Engine

The Analogical Constraint Mapping Engine (ACME; Holyoak & Thagard, 1989) is a localist connectionist model that determines analogical mappings using a parallel constraint satisfaction network. ACME posits three types of soft constraints on analogical mapping: structural, semantic, and pragmatic. According to Holyoak and Thagard (1989), "None of these constraints is absolute; rather, they provide 'pressures' (in the sense of Hofstadter, 1984) that guide the emergence of a global mapping as a consequence of numerous local decisions about element correspondences" (p. 304). This assertion distinguishes ACME from SME, which posits strict constraints that cannot be violated (e.g., one-to-one correspondence).

ACME takes as input a list of propositions representing a target and a list of propositions representing a base, plus optional semantic and pragmatic information. Each proposition is composed of a predicate (i.e., an attribute or a relation) and its arguments, which can be entities or other propositions. For example, ACME uses the propositions P1: LOVES(FRED, WILMA) and P2: KNOWS(BARNEY, P1) to represent "Barney knows Fred loves Wilma." In proposition P1, the relation LOVES takes the entities FRED and WILMA as arguments, whereas in proposition P2, the higher-order relation KNOWS takes the entity BARNEY and the proposition P1 as arguments. ACME represents functions of n arguments as relations of $n + 1$ arguments. For example, P3: GENDER(WILMA, FEMALE) represents that the gender of Wilma has the value female.

ACME automatically assigns identical predicates maximal semantic similarity and nonidentical predicates minimal semantic similarity, but also takes optional semantic information as input. For example, when comparing "Barney knows Fred loves Wilma" and "Wilma knows Barney likes Fred," the semantic similarity between the relations LOVES and LIKES can be given as input. In addition, two kinds of pragmatic information can be given as input to ACME. First, elements of the target or base can be specified as important. Secondly, correspondences between the target and the base can be specified as presumed.

Given its input, ACME constructs a network of supporting and competing hypotheses about correspondences between the target and the base. Figure 3.7 shows a schematic example from Holyoak and Thagard (1989). Numbered capital letters denote proposition identifiers, unnumbered capital letters denote predicates, lowercase letters denote entities, solid lines denote excitatory connections, and dotted lines denote inhibitory connections.

ACME creates nodes for all possible correspondences between the target and the base with the restriction that only elements of the same type can correspond. That is, propositions can only correspond to propositions (e.g., T1=B1), predicates with n arguments can only correspond to predicates with n arguments (e.g., A=M), and entities can only correspond to entities (e.g., a=m). The activation of a particular node indicates the strength of the correspondence it represents.

ACME creates symmetric excitatory connections between nodes to capture structural consistencies between possible correspondences. The structural constraint of parallel connectivity is implemented such that each potential correspondence between two propositions results in an interconnected subnetwork of mutually consistent correspondences among the elements of the propositions. For example, in Figure 3.7, the nodes representing T3=B3, C=O, a=m, and b=n mutually excite each other (the connections between C=O and a=m and between C=O and b=n are not shown). Likewise, ACME creates symmetric inhibitory connections between nodes to capture structural inconsistencies between possible correspondences. The structural constraint of one-to-one correspondence is implemented such that nodes representing different correspondences for the same element mutually inhibit each other. For example, in Figure 3.7, the nodes representing a=m and a=n mutually inhibit each other.

Semantic similarity is captured by excitatory connections from a special semantic node to all nodes representing correspondences between predicates, where the strength of each connection is proportional to the degree of semantic similarity between the predicates. For example, in Figure 3.7, the semantic node has strong excitatory connections to the nodes representing A=M and C=O because A is specified as semantically similar to M and C is specified as semantically similar to O. Likewise, pragmatic constraints are captured by strong excitatory connections from a special pragmatic node to nodes involving important

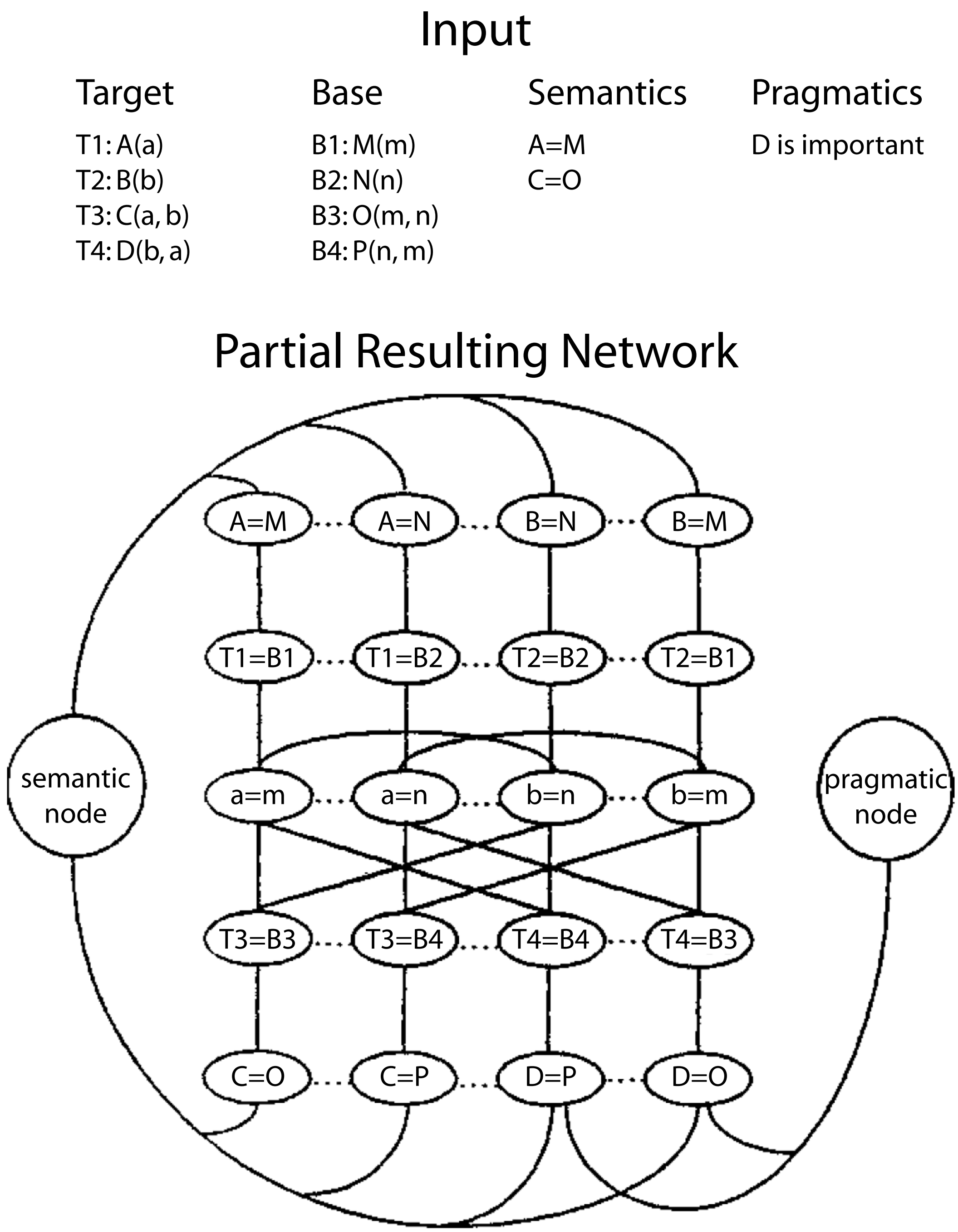

Figure 3.7: A schematic example illustrating ACME's input and the resulting network.

elements and presumed correspondences. For example, in Figure 3.7, the pragmatic node has strong excitatory connections to the nodes representing D=P and D=O because D is specified as important.

Once the network is set up, the activations of the semantic and pragmatic nodes are fixed at maximal activation and the activations of all other nodes are initialized to minimal activation. From this starting point, activation spreads through the network such that consistent correspondences support each other and inconsistent correspondences compete for activation. The dynamics of the network provide a parallel application of soft structural, semantic, and pragmatic constraints on analogical mapping. When the network reaches an equilibrium state, the activations indicate a final mapping where the most active nodes indicate the strongest correspondences between the target and the base.

ACME may be too powerful in its ability to determine mappings between structural isomorphisms that do not involve semantic similarities (Forbus, Gentner, & Law, 1994; Hummel & Holyoak, 1997). For example, given a target representing that Bill is smart and tall, Steve is smart, and Tom is timid, and a base representing that Rover is hungry and friendly, Fido is hungry, and Blackie is frisky and friendly, ACME places Bill, Steve, and Tom into correspondence with Rover, Fido, and Blackie, respectively. While Holyoak and Thagard (1989) found that subjects could determine these correspondences, Forbus et al. (1994) and Hummel and Holyoak (1997) argue that the mapping task is taken as a logical puzzle rather than an analogy. As described in Section 2.2.1, analogies involve relational similarity.

Another limitation of ACME is that it can produce final mappings that violate the structural constraint of one-to-one correspondence. This poses a problem for the inference process of copying with substitution and generation (see Section 2.2.2). A final limitation of ACME is that it does not currently address similarity comparisons.

3.3.2 Other Models of Analogy

This section briefly describes other models of analogy. These models share two characteristics. First, they do not address similarity comparisons, though it might be possible to add similarity routines to these models. Secondly, each model primarily addresses a specific aspect of analogy that is peripheral to the current work.

The Incremental Analogy Machine (IAM; Keane & Brayshaw, 1988; Keane, Ledgeway, & Duff, 1994) is intended to address the incremental nature of analogical mapping. It holds that people hypothesize correspondences and attempt to build consistent mappings from these initial starting points. IAM begins by selecting the most systematic group of interconnected elements in the base. Next, IAM selects an element in this seed group and a corresponding element in the target. From this special correspondence, called the seed match, IAM builds a consistent mapping between the seed group and the target via serial application of semantic, structural, and pragmatic constraints. If the mapping found for the seed group is accepted as optimal (e.g., more than half of the predicates in the seed group have been placed into correspondence), IAM goes on to find mappings for other groups of elements in the base such that the mappings are consistent with the mappings already found. Otherwise, the mapping found for the seed group is suboptimal and IAM backtracks and tries an alternative seed match, or if all seed matches for the current seed group have been exhausted, IAM tries an alternative seed group.

A potential problem with IAM is that Goldstone and Medin (1994) and Goldstone (1996) have demonstrated that people's comparisons are incremental, but in a different way than posited by IAM. For example, Section 2.1.6 describes data suggesting that the mapping process is driven by semantic commonalities early in processing and reflects structural constraints in time. Another limitation of IAM is that it may not be powerful enough in some respects and too powerful in others. Forbus, Ferguson, and Gentner (1994) argue that IAM may not be powerful enough to scale up to larger representations due to the fully serial nature of its processing. At the same time, Hummel and Holyoak (1997) argue that IAM may be too powerful in its ability to map unnatural analogies that do not involve semantic commonalities and that require psychologically unrealistic working memory capacity.

Learning and Inference with Schemas and Analogies (LISA; Hummel & Holyoak, 1997, 2003) is a connectionist model that stresses the importance of working memory capacity limitations. In terms of Marr's (1982) levels of analysis, LISA straddles the algorithmic and implementational levels and is intended to be biologically plausible. LISA represents predicates and entities as distributed activation patterns over semantic primitives assumed to be in long-term memory. These representations are dynamically bound into propositional structures in LISA's working memory such that only nodes with activation patterns that oscillate in synchrony are bound together (Shastri & Ajjanagadde, 1993). For example, to represent the proposition "John loves Lisa," the activation patterns of "John" and "lover" oscillate in synchrony while the activation patterns of "Lisa" and "loved" oscillate in synchrony. Alternating the activation of the two pairs in time makes it possible to unambiguously represent the whole proposition. However, LISA posits limitations on the number of different oscillating activation patterns that can be simultaneously active in working memory. Importantly, LISA only forms correspondences for propositions active in working memory.

Though a potentially powerful quality, biological plausibility limits LISA in terms of parsimony and interpretability. LISA has an elaborate control structure, complex dynamics, numerous parameters, types of nodes, and special conditions. LISA may also be limited in its ability to scale to human performance in processing analogies involving large representations (Hummel & Holyoak, 1997), though LISA can process Spellman and Holyoak's (1992) large analogy between the 1991 Persian Gulf War and World War II if provided the correspondence between Saddam Hussein and Hitler as well as a specific order in which to map subsequent propositions one at a time (Holyoak & Hummel, 2001).

Copycat (Hofstadter, 1984; Mitchell, 1993) is an agent-based model that solves proportional letter-string analogies such as "abc is to abd as mrrjjj is to *blank*" (most people answer either mrrkkk, mrrjjk, or mrrjjjj). At the heart of Copycat is the idea that representation construction and mapping are inseparable aspects of analogy (Hofstadter, 1995). Copycat's interpretations of analogies are built up from representational primitives including letters, groups of letters, and relationships between letters by stochastic computational agents acting in parallel and with no central control. This approach allows simultaneous exploration of many different interpretations of an analogy such that an optimal interpretation emerges in time. Because specific representational elements appear in the codelets that drive processing, a critical limitation of Copycat is that it is restricted to a microdomain and does not provide a domain-general account of analogy (Forbus, Gentner, Markman, & Ferguson, 1998).

3.4 Model Comparisons

Each model described in this chapter has made important contributions. Nonetheless, each is deficient in one or more respects. The models fall into three general classes: models that address both similarity and analogy (i.e., SME and CAB), models that address similarity but not analogy (i.e., MDS, the Contrast Model, RD, and SIAM), and models that address analogy but not similarity (i.e., ACME, IAM, LISA, and Copycat).

Models belonging to the first class could potentially provide a unified account of similarity judgment and analogical mapping. However, SME is insensitive to the influence of MOPs in judgments of similarity and does not account for the time course of the comparison process. CAB does not fit subjects' similarity judgments as described in Section 5.1.

Models belonging to the second class share the deficiency of not addressing analogical mapping. Both MDS models and the Contrast Model fail to capture the compositionality of mental representations and do not account for the time course of the comparison process. The central premises of RD are violated by subjects' similarity judgments as described in Section 5.1. SIAM is promising in its ability to account for many aspects of similarity judgments including MIPs and MOPs, nonmonotonicities, and the time course of the comparison process, but is not capable of processing comparisons involving systems of relations.

Models belonging to the third class share the deficiency of not addressing similarity comparisons. ACME may be too powerful in its ability to determine mappings that do not involve semantic similarities and can produce final mappings that pose a problem for the inference process of copying with substitution and generation. People's comparisons are incremental, but in a different way than posited by IAM. In addition, IAM may not scale to large analogies, but is too powerful in its ability to map unnatural analogies that do not involve semantic commonalities and that require psychologically unrealistic working memory capacity. LISA's usefulness as a model is limited by its complexity. Copycat is restricted to a microdomain and does not provide a domain-general account of analogy.

Chapter 4

Tuk-Tuk

This chapter presents Tuk-Tuk, a unified account of similarity judgment and analogical mapping. Tuk-Tuk and SIAM are closely related. Tuk-Tuk and SIAM produce identical similarity scores for comparisons that SIAM is capable of processing. However, unlike SIAM, Tuk-Tuk is capable of processing comparisons involving systems of relations and can account for benchmark phenomena of both similarity judgment and analogical mapping.

Tuk-Tuk is a localist connectionist model that determines correspondences between two structured representations via a dynamic process of interactive activation among feature, entity, and relation correspondences. Tuk-Tuk's architecture consists of a network of nodes that represent possible correspondences between compared representations. The activation of a particular node indicates the strength of the correspondence it represents.

Like other connectionist systems (e.g., CAB, SIAM, and ACME), activation spreads through Tuk-Tuk's network along excitatory and inhibitory connections between nodes. Nodes representing consistent correspondences excite each other while nodes representing inconsistent correspondences inhibit each other. Tuk-Tuk displays desirable properties stereotypically associated with both symbolic and connectionist architectures by pairing structured representations with a parallel constraint satisfaction approach to comparison.

Tuk-Tuk takes as input two structured representations. Tuk-Tuk's output is a set of

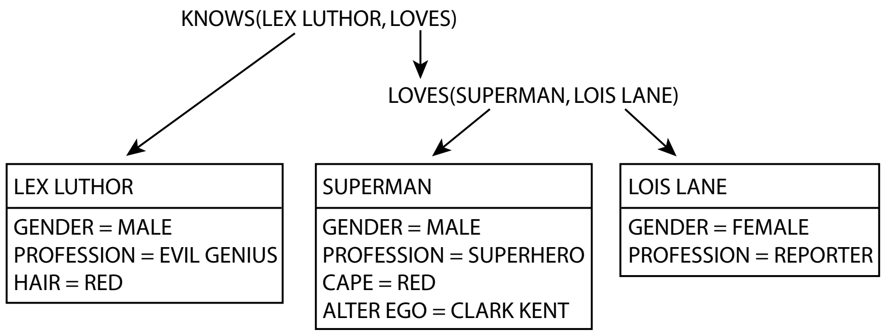

Figure 4.1: A representation of a scenario from the movie Superman.

correspondences between the representations and a score indicating the overall similarity or analogousness of the representations. Before discussing how Tuk-Tuk arrives at this output, I will discuss Tuk-Tuk's input.

4.1 Knowledge Representation

The input to Tuk-Tuk consists of a target representation and a base representation. Figure 4.1 shows a representation of a scenario from the movie Superman, in which Lex Luthor knows Superman loves Lois Lane. Tuk-Tuk's representations are composed of entities (e.g., Lex Luthor), features that constitute entities (e.g., gender = male), and predicates that take entities or other predicates as arguments (e.g., knows(Lex Luthor, loves)). A feature consists of a dimension and a value for that dimension. Like ACME, Tuk-Tuk represents functions of n arguments as relations of $n + 1$ arguments.

Features, attributes, and relations can represent logically equivalent knowledge. For example, the fact that Lex Luthor is male can be represented by the gender = male feature of the entity Lex Luthor, by the attribute male(Lex Luthor), or by the relation gender(Lex Luthor, male). While these representations are logically equivalent, Tuk-Tuk allows for each representation because they are psychologically distinct. A feature represents a property that is intrinsic to an entity and cannot be assessed independent of that entity. Unlike a feature, an attribute is an independent semantic unit that describes its argument (e.g., MALE describes LEX LUTHOR in MALE(LEX LUTHOR)). However, an attribute does not represent which aspect of an entity it describes (e.g., MALE(LEX LUTHOR) represents that Lex Luthor is male, not that Lex Luthor's gender is male). In contrast, the relation GENDER(LEX LUTHOR, MALE) represents a gender relationship between the entity LEX LUTHOR and the entity MALE.

4.2 Processing

Tuk-Tuk's architecture consists of a network of nodes that represent hypothetical feature-to-feature, entity-to-entity, and relation-to-relation correspondences between the target and the base. The activation of a particular node indicates the strength of the correspondence it represents. Node activations range from 0 to 1, where 1 indicates maximal certainty that two elements correspond, 0 indicates maximal certainty that two elements do not correspond, and 0.5 indicates maximal uncertainty.

Each feature-to-feature node represents a hypothesis that two features correspond. Tuk-Tuk only creates feature-to-feature nodes for features that have the same dimension. For example, when comparing a red square to a blue circle, Tuk-Tuk creates a node for the hypothesis that COLOR = RED corresponds with COLOR = BLUE, but does not create a node for the hypothesis that COLOR = RED corresponds with SHAPE = CIRCLE. If d is the number of dimensions represented both in the target and in the base, the number of feature-to-feature nodes in Tuk-Tuk's network is

$$F = \sum_{i=1}^{d} F_i, \tag{4.1}$$

where F_i is the number of features in the target describing dimension i times the number of features in the base describing dimension i.

Entity-to-entity nodes each represent a hypothesis that two entities correspond. If there are m entities in the target and n entities in the base, the number of entity-to-entity nodes in Tuk-Tuk's network is $E = mn$.

Each relation-to-relation node represents a hypothesis that two relations correspond. Tuk-Tuk only creates nodes for relations that have the same arity (like ACME) and types of arguments. For example, when comparing the movies Spider-Man and Superman, Tuk-Tuk creates a node for the hypothesis that LOVES(SPIDER-MAN, MARY JANE) corresponds with LOVES(SUPERMAN, LOIS LANE), but does not create a node for the hypothesis that LOVES(SPIDER-MAN, MARY JANE) corresponds with GIVES(LEX LUTHOR, KRYPTONITE, SUPERMAN) because they have different arities, and does not create a node for the hypothesis that LOVES(SPIDER-MAN, MARY JANE) corresponds with KNOWS(LEX LUTHOR, LOVES) because they have different types of arguments. If there are j relations in the target and k relations in the base, the number of relation-to-relation nodes in Tuk-Tuk's network is $R \leq jk$. Because Tuk-Tuk only creates nodes for relations that have the same arity and types of arguments, R is typically much less than jk.

In addition to their activations, feature-to-feature and relation-to-relation nodes have match values that represent semantic similarity between their associated features or relations. If the features or relations are identical, the match value is 1. Otherwise, the match value is set to a model parameter that is less than 1 and greater than or equal to 0. For example, a node placing LOVES(SPIDER-MAN, MARY JANE) into correspondence with LOVES(SUPERMAN, LOIS LANE) has a match value of 1, and a node placing PROFESSION = ACTRESS into correspondence with PROFESSION = REPORTER has a match value less than 1 and greater than or equal to 0.

Activation flows through Tuk-Tuk's network in three ways: nodes representing consistent correspondences excite each other, nodes representing inconsistent correspondences inhibit each other, and match values excite feature-to-feature and relation-to-relation nodes. Excitatory and inhibitory connections between nodes in the network are established according to the structure of the representations being compared. Match values are established according to the semantics of the representations being compared. Figure 4.2 illustrates excitatory connections, inhibitory connections, and match values in Tuk-Tuk's network.

Correspondences between entities are excited by correspondences between their features. For example, the correspondence between Spider-Man and Superman is excited by the correspondence between profession = superhero and profession = superhero (see Figure 4.2). At the same time, correspondences between features are excited by correspondences between the entities they describe. For example, the correspondence between profession = actress and profession = reporter is excited by the correspondence between Mary Jane and Lois Lane. Correspondences between relations are excited by correspondences between their arguments, which can be entities or other relations. For example, the correspondence between loves(Spider-Man, Mary Jane) and loves(Superman, Lois Lane) is excited by the correspondence between Spider-Man and Superman and the correspondence between Mary Jane and Lois Lane. At the same time, correspondences between entities or relations are excited by correspondences between the relations that take them as arguments. For example, the correspondence between Spider-Man and Superman and the correspondence between Mary Jane and Lois Lane are excited by the correspondence between loves(Spider-Man, Mary Jane) and loves(Superman, Lois Lane).

There are also excitatory and inhibitory connections between feature-to-feature nodes, between entity-to-entity nodes, and between relation-to-relation nodes. These connections are inhibitory if the two nodes taken together place an element in one representation into correspondence with two elements in the other representation, and excitatory otherwise. For example, the node placing Spider-Man into correspondence with Superman has an inhibitory connection with the node placing Mary Jane into correspondence with Superman and an excitatory connection with the node placing Mary Jane into correspondence with Lois Lane. Although the issue does not arise in Figure 4.2, feature-

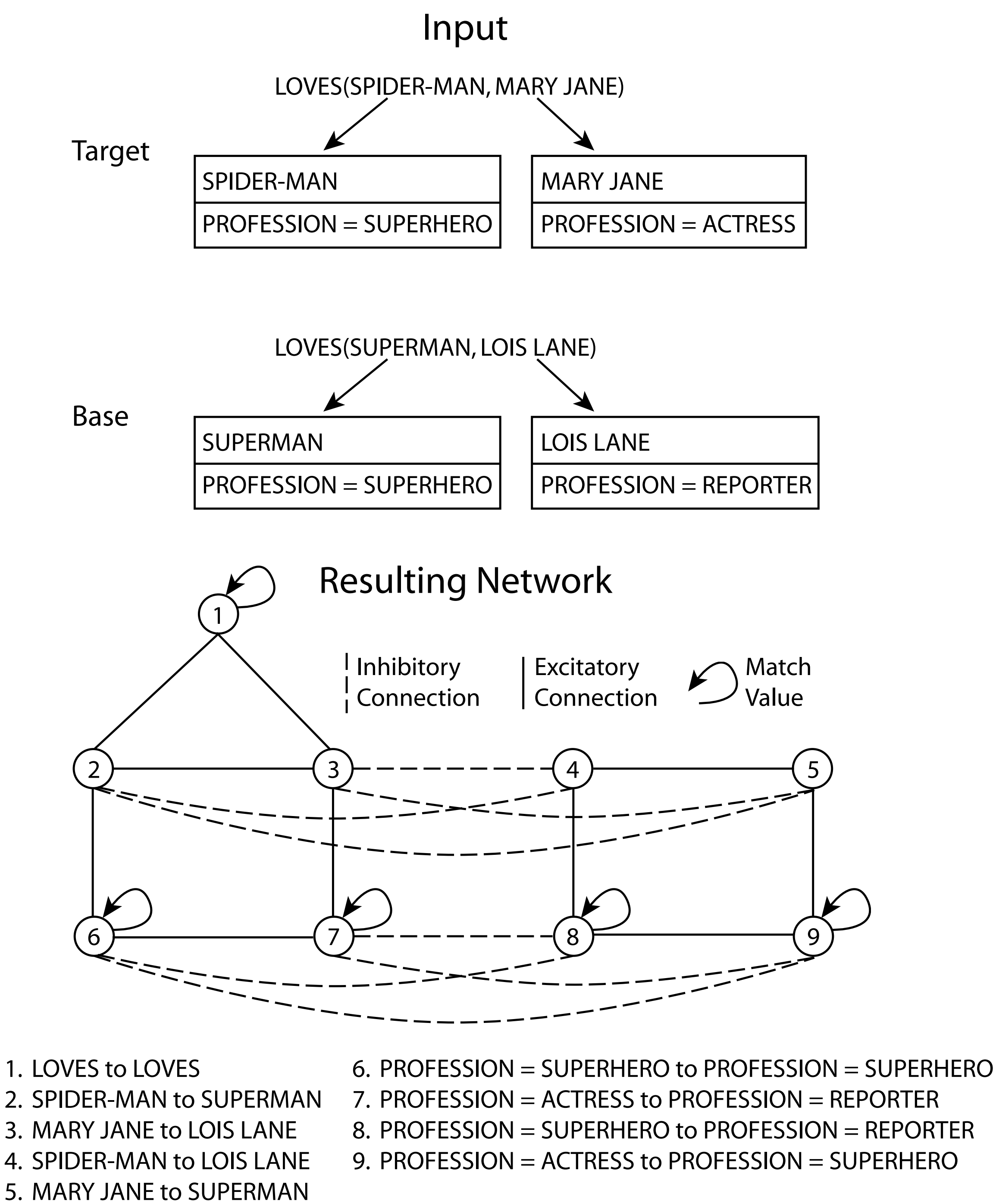

Figure 4.2: Tuk-Tuk's input and the resulting network for a simple comparison between the movies Spider-Man and Superman.

to-feature nodes only excite or inhibit other feature-feature nodes that describe the same dimension. For example, a node placing two genders into correspondence does not directly influence a node placing two professions into correspondence. Also, relation-to-relation nodes only excite or inhibit other relation-to-relation nodes if all four associated relations have the same arity and types of arguments. For example, a node placing relations of arity two into correspondence does not excite a node placing relations of arity three into correspondence.

In addition to being influenced by other nodes, feature-to-feature and relation-to-relation nodes are excited by their match values. For example, the node placing LOVES(SPIDER-MAN, MARY JANE) into correspondence with LOVES(SUPERMAN, LOIS LANE) is excited by its match value of 1.

Once Tuk-Tuk's network is established, processing consists of updating each node's activation according to its match value (except entity-to-entity nodes, which do not have match values) and the activations of connected nodes. Node activations are updated for a specified number of time steps. Before the first time step, all node activations are initialized to 0.5, indicating maximal uncertainty for all correspondences. Each time step, nodes and match values "vote" to determine the activation of the nodes they excite or inhibit. Each node computes a weighted average of these votes, and updates its activation accordingly. The computational complexity of updating the network each time step is

$$O\left(\sum_{i=1}^{d} F_i^2 + E^2 + R^2\right) \tag{4.2}$$

since all feature-to-feature nodes describing the same dimension are interconnected, all entity-to-entity nodes are interconnected, and all relation-to-relation nodes are interconnected.

Formally, let $A_{i,t}$ be the activation of node i at time t. Node activations are updated according to

$$A_{i,t+1} = (1 - L)A_{i,t} + LM_i, \tag{4.3}$$

where L is a model parameter that determines the rate of change for node activations and

$$M_i = \frac{\sum_{j=1}^{n} W_{ji} V_{ji}}{\sum_{j=1}^{n} W_{ji}} \tag{4.4}$$

is the weighted average of votes from node i's match value and nodes connected to node i. V_{ji} is node j's vote for node i's activation, W_{ji} is the weight of this vote, and n is the number of nodes connected to node i (if node i is a feature-to-feature or relation-to-relation node, n includes its match value). The value of W_{ji} is a model parameter that depends on the types of nodes j and i (see Table 4.1 for a complete list of model parameters). For example, if node j is an entity-to-entity node and node i is a feature-to-feature node, then W_{ji} is given by the parameter entity-to-feature-wt.

For excitatory connections,

$$V_{ji} = \begin{cases} A_i + (A_j - 0.5)(1 - A_i), & \text{if } A_j > 0.5 \\ A_i - (0.5 - A_j)A_i, & \text{if } A_j \leq 0.5. \end{cases} \tag{4.5}$$

For inhibitory connections,

$$V_{ji} = \begin{cases} A_i + (0.5 - A_j)(1 - A_i), & \text{if } A_j < 0.5 \\ A_i - (A_j - 0.5)A_i, & \text{if } A_j \geq 0.5. \end{cases} \tag{4.6}$$

Both excitatory and inhibitory connections from node j can increase or decrease the activation of node i. If node j's activation is above 0.5, node i's activation increases if the connection is excitatory and decreases if it is inhibitory. If node j's activation is below 0.5, node i's activation decreases if the connection is excitatory and increases if it is inhibitory. The degree to which V_{ji} differs from node i's activation depends on how far the activations of nodes j and i are from the minimum or maximum activation. The more extreme A_j is,

Table 4.1: Tuk-Tuk's parameters.

`num-time-steps` = number of time steps activations are updated. No default

L = rate of change for activations (0-1). Default = 1

`feature-mismatch-value` = match value (0-1) for non-identical features. Default = 0

`feature-mismatch-wt` = weight of feature mismatch on feature-to-feature node. Default = 1

`feature-match-wt` = weight of feature match on feature-to-feature node. Default = 1

`feature-to-feature-inhibit-wt` = weight of inconsistent feature-to-feature nodes on each other. Default = 1

`feature-to-feature-excit-wt` = weight of consistent feature-to-feature nodes on each other. Default = 1

`feature-to-entity-wt` = weight of feature-to-feature node on consistent entity-to-entity node. Default = 1

`entity-to-feature-wt` = weight of entity-to-entity node on consistent feature-to-feature node. Default = 1

`entity-to-entity-inhibit-wt` = weight of inconsistent entity-to-entity nodes on each other. Default = 1

`entity-to-entity-excit-wt` = weight of consistent entity-to-entity nodes on each other. Default = 1

`entity-to-relation-wt` = weight of entity-to-entity node on consistent relation-to-relation node. Default = 1

`relation-to-entity-wt` = weight of relation-to-relation node on consistent entity-to-entity node. Default = 1

`relation-mismatch-value` = match value (0-1) for non-identical relations. Default = 0

`relation-mismatch-wt` = weight of a relation mismatch on relation-to-relation node. Default = 1

`relation-match-wt` = weight of a relation match on relation-to-relation node. Default = 1

`relation-to-relation-inhibit-wt` = weight of inconsistent relation-to-relation nodes on each other. Default = 1

`relation-to-relation-excit-wt` = weight of consistent relation-to-relation nodes on each other. Default = 1

`relation-to-relation-wt` = weight between relation correspondences and argument correspondences if arguments are relations. Default = 1

`threshold` = minimum activation for correspondence. Default = 0.6.

the more V_{ji} differs from A_i, whereas the more extreme A_i is, the less V_{ji} differs from A_i.

At any stage in processing, overall similarity between the target and base can be calculated as a function of match values weighted by current node activations:

$$\text{similarity} = \frac{\sum_{i=1}^{n} A_i \text{match-value}_i}{\sum_{i=1}^{n} A_i}, \tag{4.7}$$

where n is the total number of feature-to-feature and relation-to-relation nodes in Tuk-Tuk's network. Similarity is normalized and falls between 0 and 1. Matching and mismatching features and relations respectively increase and decrease similarity to a greater extent if they are placed into strong correspondence than if they are placed into weak correspondence.

To generate a mapping in which elements either correspond or do not correspond between the target and the base, Tuk-Tuk applies a filter to the current node activations. Each node in Tuk-Tuk's network yields a correspondence between its associated elements only if the node's activation is greater than the activation of any other node associated with either of its elements, and the node's activation is greater than a threshold given by a model parameter. This limits any element in one representation to corresponding to at most one element in the other representation, and requires a given degree of certainty before generating a correspondence.

Chapter 5

Evaluating Tuk-Tuk

In this chapter, Tuk-Tuk's performance is tested using a broad set of simulations, including simulations of a new behavioral study. Although Table 4.1 lists 20 model parameters that could potentially be varied, Tuk-Tuk generally does not require parameter adjustments to function properly. The majority of the simulations were conducted using default parameters. There are three exceptions: relation-to-relation-inhibit-wt is varied in a study of Rutherford's analogy between the atom and the solar system (see Section 5.2), relation-to-entity-wt and entity-to-feature-wt are varied in a study of literal similarity comparisons (see Section 5.3.7), and relation-to-relation-inhibit-wt and feature-to-feature-inhibit-wt are varied in a study of the "Karla the hawk" analogy (see Section 5.3.12). Importantly, each parameter variation has psychological meaning and is analyzed over the full range of the parameter.

5.1 Similarity Judgment: Distinguishing Tuk-Tuk from RD, SME, and CAB

This section presents an experiment and simulations that distinguish Tuk-Tuk from RD, SME, and CAB in its ability to account for patterns of similarity ratings. The experiment was designed to answer several questions. First, does a given transformation have a fixed, non-positive influence on similarity as predicted by RD? Secondly, are MOPs irrelevant to similarity as predicted by SME? Thirdly, if MOPs do influence similarity, does CAB or Tuk-Tuk capture the nature of this influence?

One benefit of the experiment is that it provides a single empirical test that encompasses each model under consideration. Another benefit is that the stimuli used in the experiment involve simple geometric entities that have spatially integrated but conceptually separable feature dimensions such as color and shape. This stands in contrast to stimuli in which feature dimensions are associated with specific spatial regions of entities (for example, see Figure 2.6). A potential problem with such stimuli is that visual alignment may be confounded with conceptual alignment.

To test RD, SME, CAB, and Tuk-Tuk, subjects were asked to rate the similarity of two configurations each consisting of a pair of objects. Objects varied in color and shape in the original study and color and texture in a replication. Objects within a pair were configured such that either one object was above the other object or one object was beside the other object.

To create one pair of objects, two different colors, two different shapes (or textures), and a spatial relation were randomly selected. To create the second pair of objects, the colors, shapes (or textures), and relation in the first configuration were independently transformed. The transformations were designed to test whether a given transformation has a fixed, non-positive influence on similarity, and to systematically vary the number of MIPs and MOPs.

5.1.1 Subjects

A total of 116 subjects participated either for an \$8.00 payment or to fulfill a course requirement at the University of Texas at Austin. Fifty-eight subjects participated in a study involving color and shape, and fifty-eight in a study involving color and texture.

5.1.2 Materials

Subjects were shown 162 displays on a 43 cm wide × 34 cm high LCD screen. Each display contained two pairs of objects. In the original study, objects varied in color (green, red, blue, or yellow) and shape (triangle, circle, square, or star). In the replication, square objects varied in color (green, red, blue, or yellow) and texture (bark, carpet, pasta, or bubbles). Each individual object was approximately 3 cm wide × 3 cm high. Objects within each pair were configured such that either one object was above the other object or one object was beside the other object. The distance between objects within a pair was 1 cm. Pairs were randomly positioned on the screen subject to the constraint that the distance between the pairs was 11 cm.

5.1.3 Design

On each trial, a base pair was constructed by randomly selecting two different colors, two different shapes (or textures), and a spatial relation. A target pair was constructed by selectively altering the features and relation of the base pair.

On half the trials, the relation between the objects was not altered; on the remaining half, the relation was different (e.g., if the objects in the base pair were one above the other, the objects in the target pair were one beside the other). When the relation was different, the top object in one pair was randomly selected as either the left or right object in the other pair.

Each feature dimension of the base pair was altered in one of nine ways, as illustrated in Figure 5.1. A pair's respective values on a particular dimension can be abstractly represented by letters. For example, a pair with shapes represented by AB has shape A for one object and shape B for the other object. If AB denotes the base pair's respective values on a particular dimension, then the methods used to transform each dimension were: AB→AB (no change), AB→BA (switch values), AB→AA (copy one value), AB→BB (copy the other value), AB→AC (replace one value), AB→CB (replace the other value), AB→CA

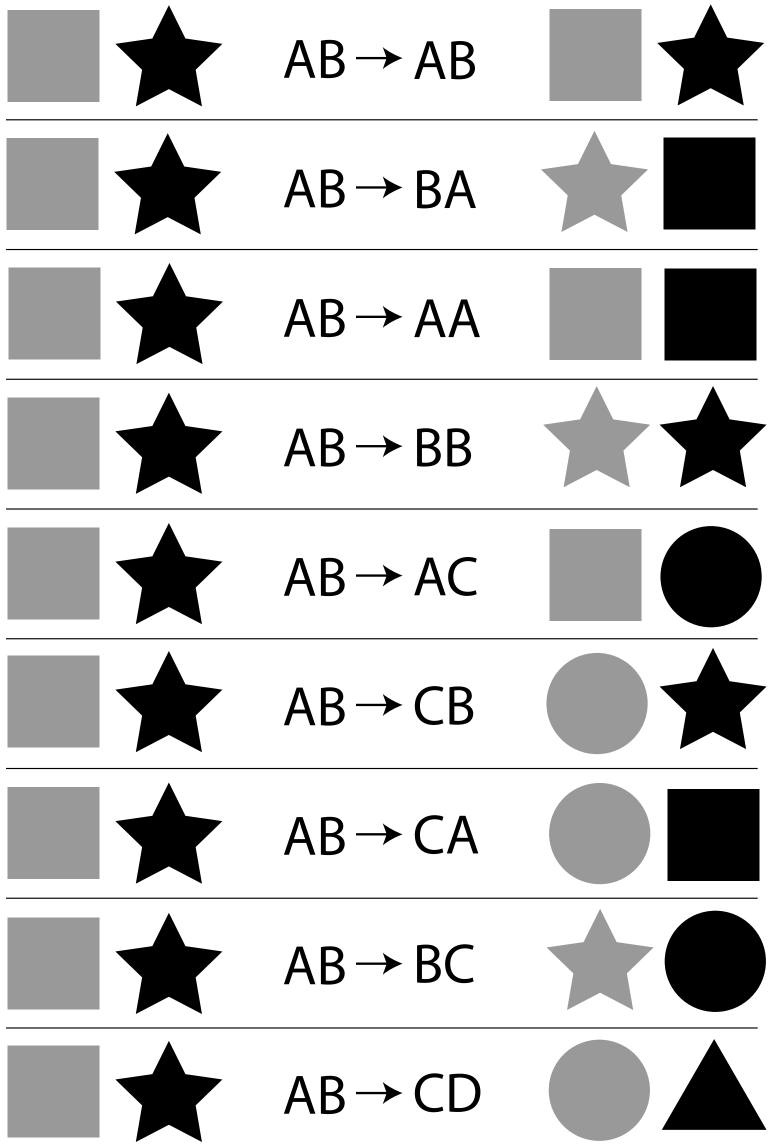

Figure 5.1: Methods for altering feature dimensions. The target pairs (right column) were constructed by altering the shapes of the base pair (left column) according to each transformation (middle column).

(replace one value and switch values), AB→BC (replace the other value and switch values), AB→CD (replace both values). Each feature dimension and the relation were transformed independently, creating a total of 162 unique trials (9 methods of changing one dimension $\times$ 9 methods of changing the other dimension $\times$ 2 methods for changing the relation).

To control for the possibility that the absolute positions of objects or spatial relations between pairs might act as cues for mapping the objects, pairs were randomly positioned on the screen subject to the constraint that the distance between the pairs was always the same. Thus the only consistent spatial cue for mapping was the location of an object relative to the other object in its pair.

5.1.4 Procedure

Each trial began with the simultaneous display of the base pair and the target pair. The subjects' task was to rate the pairs' similarity on a scale from one (low similarity) to six (high similarity) that was displayed at the bottom of the screen. It was emphasized that subjects should compare the pairs and rate the similarity between the pairs. After subjects submitted a rating by clicking on the appropriate button, the screen was erased and subjects proceeded to the next trial. Each subject was presented with all 162 trials in randomized order.

5.1.5 Results

The primary data used to test the models are the subjects' similarity ratings for each method of changing one dimension when the method of changing the other dimension is AB→AB (i.e., the values are not changed) or AB→BA (i.e., the values are switched). Note that the methods AB→AA and AB→BB, AB→AC and AB→CB, and AB→CA and AB→BC are functionally equivalent when the method of changing the other dimension is symmetric (i.e., AB→AB or AB→BA). Thus ratings for these methods are collapsed and labeled by the former method (e.g., AB→AA denotes both AB→AA and AB→BB).

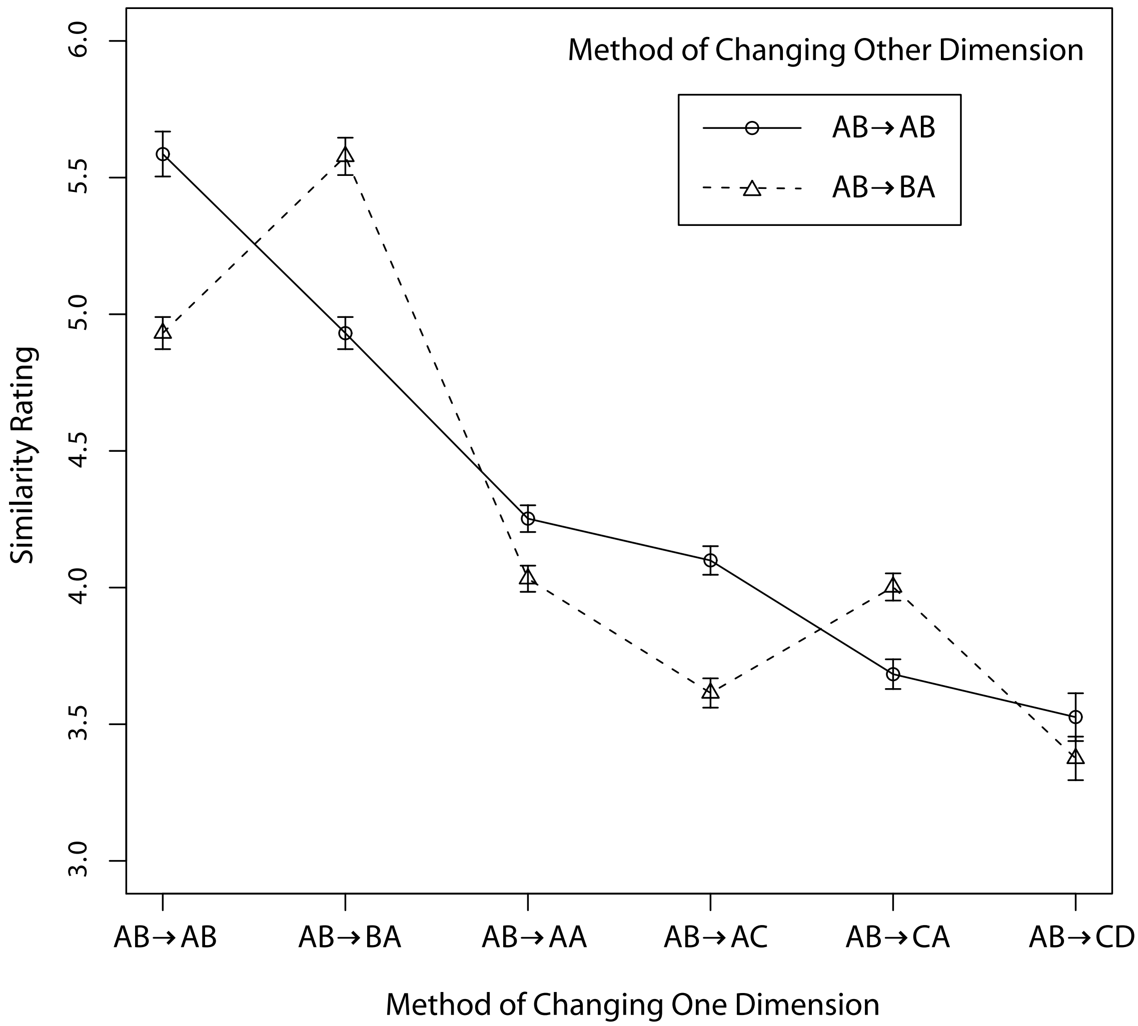

Figure 5.2: Mean similarity ratings for each method of changing one dimension when the method of changing the other dimension is AB→AB or AB→BA. Feature dimensions are color and shape. Error bars denote standard errors.

The data for the study involving color and shape are shown in Figure 5.2. All pairwise differences between the methods of changing one dimension when the method of changing the other dimension is AB→AB are significant ($p < .001$ using Tukey's HSD) except AB→AA and AB→AC, and AB→CA and AB→CD. All pairwise differences when the method of changing the other dimension is AB→BA are significant ($p < .001$ using Tukey's HSD) except AB→AA and AB→CA, and AB→AC and AB→CD.

The data for the version involving color and texture are shown in Figure 5.3. All pairwise differences between the methods of changing one dimension when the method of changing the other dimension is AB→AB are significant ($p < .001$ using Tukey's HSD) except AB→BA, AB→AA, and AB→AC, and AB→CA and AB→CD. All pairwise differences when the method of changing the other dimension is AB→BA are significant ($p < .001$ using Tukey's HSD) except AB→AB and AB→AA, AB→AA and AB→CA, and AB→AC and AB→CD.

5.1.6 Comparison to model predictions

RD has difficulty accounting for these data. Post hoc weighting of transformations allows RD to fit either the pattern when the method of changing the other dimension is AB→AB, or the pattern when the method of changing the other dimension is AB→BA, but not both. No weighting of transformations can simultaneously fit both patterns of data. There are two reasons for this failure. First, a given transformation does not have a fixed influence on similarity in all contexts. For example, AB→CA decreases similarity more than AB→AC when the method of changing the other dimension is AB→AB, but AB→CA decreases similarity less than AB→AC when the method of changing the other dimension is AB→BA. Secondly, a given transformation can have a negative influence on similarity in some cases and a positive influence on similarity in others. For example, AB→BA decreases similarity when the method of changing the other dimension is AB→AB, but increases similarity when the method of changing the other dimension is AB→BA.

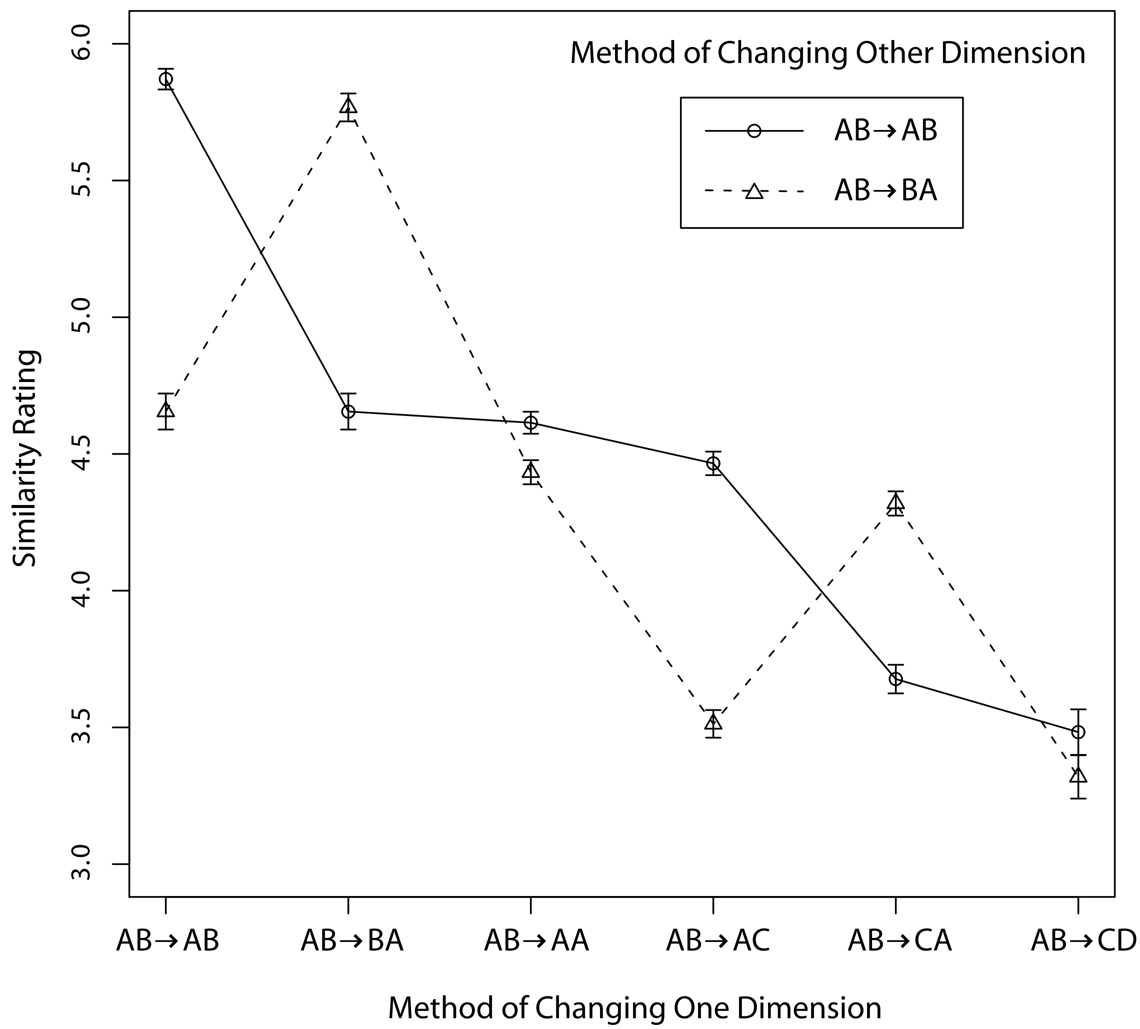

Figure 5.3: Mean similarity ratings for each method of changing one dimension when the method of changing the other dimension is AB→AB or AB→ BA. Feature dimensions are color and texture. Error bars denote standard errors.

Structure-mapping accounts such as SME, CAB, and Tuk-Tuk differ from transformational accounts such as RD in two fundamental ways. First, structure-mapping accounts specify the process by which correspondences are determined, whereas transformational accounts do not. Secondly, structure-mapping accounts define similarity in terms of matching or mismatching representational constituents that correspond, whereas transformational accounts require additional processing to determine the transformations that distort one representation into the other. It might be possible to fit the data using a weighted transformational account that uses structure mapping to determine correspondences, but because a structure-mapping account can alone account for the data, the additional suppositions of transformational accounts are superfluous.

Structure-mapping accounts are alike in that correspondences between representational constituents affect judgments of similarity, but they differ in how correspondences emerge from the comparison process. Unlike Tuk-Tuk and CAB, SME ignores matching features that are inconsistent with the dominant mapping (i.e., MOPs). According to SME, similarity is a monotonically increasing function of the number of MIPs between compared items. Thus a critical test of SME is whether the number of MIPs predicts the ordinal relationship between methods of changing one dimension when the method of changing the other dimension is AB→AB. The ordinal relationship predicted by SME is shown in Figure 5.4. SME fits the subjects' similarity ratings with the exception of method AB→BA. For example, when comparing a red square above a blue circle to a red circle above a blue square, SME places the red square into correspondence with the red circle and the blue circle into correspondence with the blue square. According to SME, the two matching colors that correspond (MIPs) increase the similarity of the pairs, but the two matching shapes that do not correspond (MOPs) do not increase similarity. The discrepancy between SME's predictions and the data is due to subjects' sensitivity to MOPs as well as MIPs.

Both Tuk-Tuk and CAB are sensitive to MOPs as well as MIPs. While MIPs and MOPs increase similarity, inconsistent MIPs and MOPs indirectly *decrease* similarity

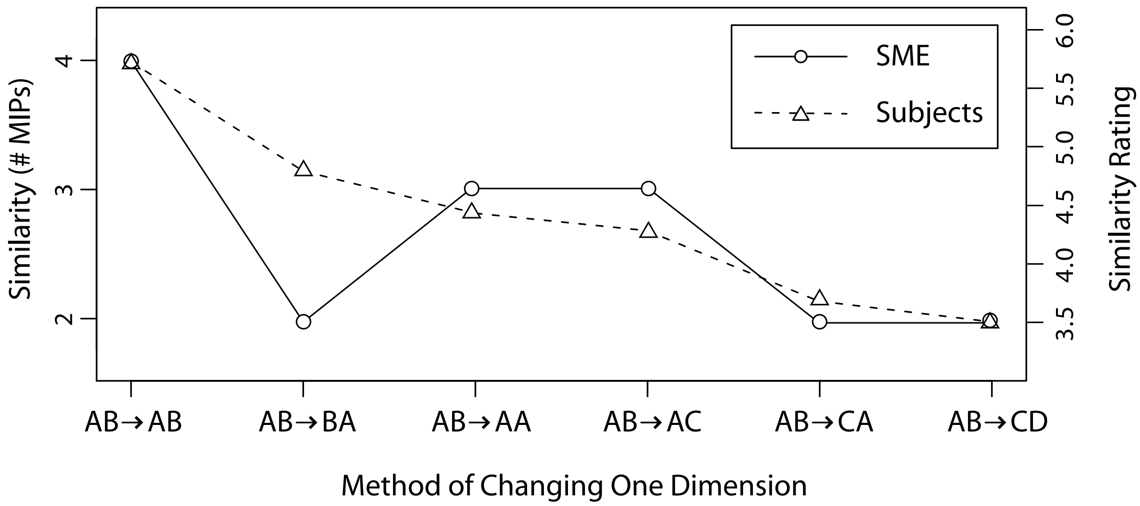

Figure 5.4: SME's predicted ordinal relationship between methods of changing one dimension when the method of changing the other dimension is AB→AB. Subjects' mean similarity ratings combined over both stimulus sets are shown on a separate scale.

by competing for activation. For example, when comparing a red square above a blue circle to a red circle above a blue square, the matching shapes compete with the matching colors for activation because the shape correspondences are inconsistent with the color correspondences. The shapes vote for placing the red square into correspondence with the blue square and the blue circle into correspondence with red circle, while the colors vote for placing the red square into correspondence with the red circle and the blue circle into correspondence with the blue square. These correspondences are inconsistent because taken together they constitute a many-to-one mapping between the pairs.

Whereas SME's adherence to one-to-one correspondence is strict, Tuk-Tuk treats one-to-one correspondence as a soft constraint on node activations.[1] As a result, and because Tuk-Tuk allows for different degrees of correspondence (unlike SME's all-or-nothing notion of correspondence), both MIPs and MOPs influence similarity, with MIPs having a stronger influence than MOPs.

[1]While one-to-one correspondence is a soft constraint on Tuk-Tuk's node activations, Tuk-Tuk generates strict one-to-one mappings for the purpose of analogical inference.

Tuk-Tuk and CAB differ in the degree to which competition between inconsistent MIPs and MOPs negatively influences similarity. Because CAB's adherence to one-to-one mappings is stricter than Tuk-Tuk's, competition in CAB has a stronger negative influence on similarity than it does in Tuk-Tuk. Also, whereas semantic commonalities only influence the initial activations of CAB's network, match values compensate for competition between MIPs and MOPs by continually exciting Tuk-Tuk's network. As a result, competition between MIPs and MOPs decreases similarity more in CAB than in Tuk-Tuk.

CAB fits the ordinal relationship between methods of changing one dimension when the method of changing the other dimension is AB→AB with the exception of the relationship between AB→BA and AB→AA. From highest to lowest similarity, CAB generates the following order: AB→AB, AB→AA, AB→BA, AB→AC, AB→CA, AB→CD. CAB has difficulty with the relationship between AB→BA and AB→AA because it overestimates competition between the AB→AB method of changing one dimension and the AB→BA method of changing the other dimension. This difficulty is prevalent throughout CAB's parameter space.

Tuk-Tuk predicts the data using its default parameters (see Table 4.1). Figure 5.5 illustrates the representations used in the simulations. From highest to lowest similarity, Tuk-Tuk generates the following order when the method of changing the other dimension is AB→AB: AB→AB, AB→BA, AB→AA, AB→AC, AB→CA, AB→CD. Tuk-Tuk generates the following order when the method of changing the other dimension is AB→BA: AB→BA, AB→AB, AB→AA, AB→CA, AB→AC, AB→CD. Tuk-Tuk's ability to predict the data arises because (unlike RD) it is sensitive to subjective correspondences between compared items, (unlike SME) it is sensitive to MOPs as well as MIPs, and (unlike CAB) it does not overestimate competition between inconsistent MIPs and MOPs. Tuk-Tuk predicts both patterns of data without post hoc parameter fitting.

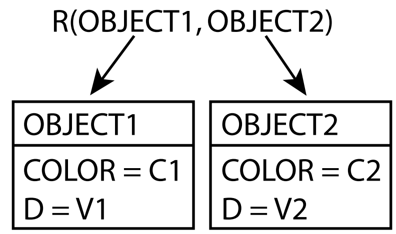

Figure 5.5: An abstract example of Tuk-Tuk's representations for simulating the influence of MIPs and MOPs on similarity. Depending on the particular stimulus, R is either ABOVE or BESIDE, C1 and C2 are colors, D is either SHAPE or TEXTURE, and V1 and V2 are either shapes or textures.

5.1.7 Conclusions

These results are consistent with the hypothesis that correspondences between representational constituents modulate similarity judgments. The data are inconsistent with transformational accounts of similarity because a given transformation does not have a fixed, non-positive influence on perceived similarity. Within structure-mapping accounts of similarity, only Tuk-Tuk fully captures subjects' similarity judgments. Inconsistent with SME's predictions, MOPs affect rated similarity. Inconsistent with CAB's predictions, competing MIPs and MOPs do not drastically decrease similarity.

5.2 Analogical Mapping: The Atom and the Solar System

Tuk-Tuk is applicable to both similarity and analogy. This section presents a simulation highlighting Tuk-Tuk's promise as a model of analogical mapping. This simulation, borrowed from Falkenhainer et al. (1989), applies Tuk-Tuk to Rutherford's analogy between the atom and the solar system (see Table 5.1).

With relation-to-relation-inhibit-wt ≥ 1.4 and default values for all other parameters, Tuk-Tuk correctly places the nucleus into correspondence with the Sun and the electrons into correspondence with the planets. This simulation also demonstrates that Tuk-Tuk is consistent with the benchmark of systematicity in using higher-order relations

Table 5.1: Tuk-Tuk's representations for Rutherford's analogy between the atom and the solar system.

Atom	Solar System
NUCLEUS	SUN
ELECTRONS	PLANETS
OPPOSITE SIGN(NUCLEUS, ELECTRONS)	GRAVITY BETWEEN(SUN, PLANETS)
ATTRACTS(NUCLEUS, ELECTRONS)	ATTRACTS(SUN, PLANETS)
CAUSES(OPPOSITE SIGN, ATTRACTS)	CAUSES#1(GRAVITY BETWEEN, ATTRACTS)
LARGER(NUCLEUS, ELECTRONS)	LARGER(SUN, PLANETS)
REVOLVES AROUND(ELECTRONS, NUCLEUS)	REVOLVES AROUND(PLANETS, SUN)
	AND(LARGER, ATTRACTS)
	CAUSES#2(AND, REVOLVES AROUND)
	HOTTER(SUN, PLANETS)

to disambiguate possible correspondences. For example, Tuk-Tuk places the sign difference between the nucleus and the electrons (which causes the nucleus to attract the electrons) into correspondence with gravity between the Sun and the planets (which causes the Sun to attract the planets), whereas the difference in temperature between the Sun and the planets is irrelevant to the analogy.

A parameter study reveals the importance of the structural constraint of one-to-one correspondence. In mapping Rutherford's analogy, Tuk-Tuk is sensitive to the parameter relation-to-relation-inhibit-wt, which controls inhibition between inconsistent relation-to-relation correspondences that taken together place a relation in one representation into correspondence with two relations in the other representation. Using default values for all other parameters, Tuk-Tuk's behavior changes abruptly between relation-to-relation-inhibit-wt = 1.3 and relation-to-relation-inhibit-wt = 1.4. With relation-to-relation-inhibit-wt ≤ 1.3, Tuk-Tuk places into correspondence CAUSES and CAUSES#2, NUCLEUS and PLANETS, and ELECTRONS and SUN. With relation-to-relation-inhibit-wt ≥ 1.4, Tuk-Tuk generates the standard interpretation of the analogy. Thus, when the influence of one-to-one

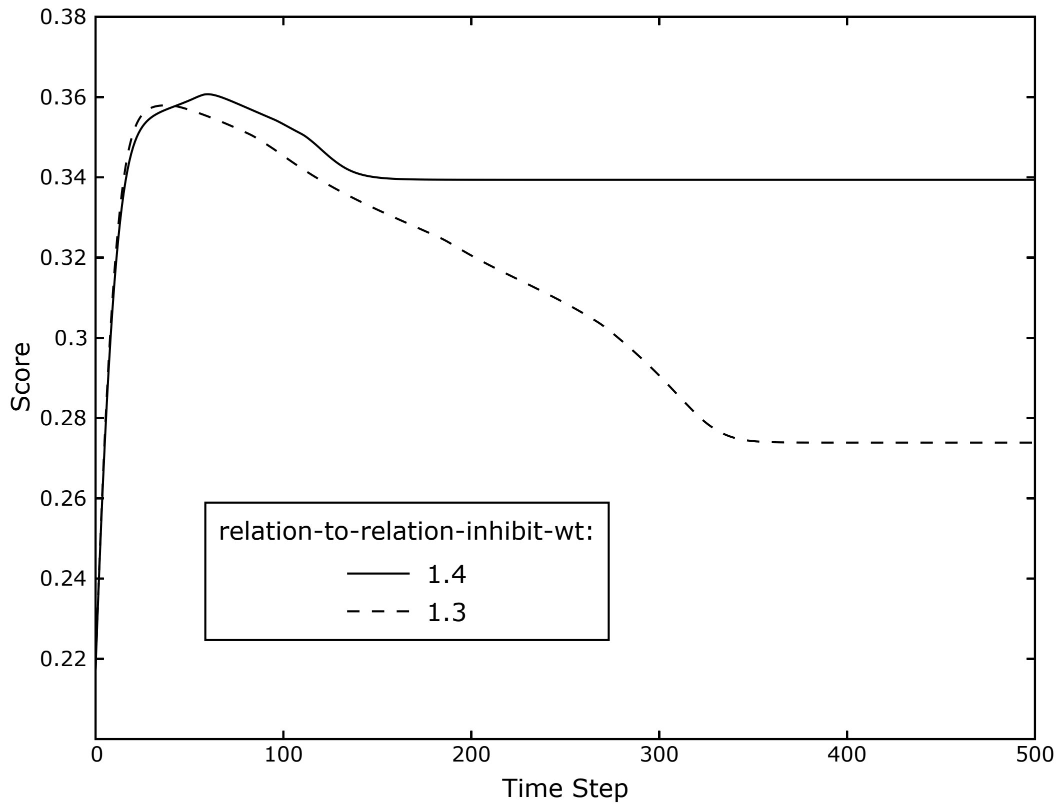

Figure 5.6: Scores over time for two interpretations of Rutherford's analogy.

correspondence is weak, Tuk-Tuk generates a nonstandard interpretation, but when the influence of one-to-one correspondence is strong, Tuk-Tuk generates the standard interpretation. Importantly, Tuk-Tuk gives a higher score for the standard interpretation than the nonstandard interpretation (see Figure 5.6).

5.3 Addressing Benchmark Phenomena

While the simulations described above highlight Tuk-Tuk's promise as a unified account of similarity judgment and analogical mapping, Tuk-Tuk's ability to account for the full range of empirical phenomena described in Chapter 2 and listed in Table 5.2 needs to be verified. This section presents simulations that address each of these benchmark phenomena.

Table 5.2: Benchmark phenomena addressed by Tuk-Tuk.

Similarity Judgment:	1. Structured Representations 2. Matches in Place and Matches out of Place 3. Alignable and Nonalignable Differences 4. Nonmonotonicity 5. Asymmetry 6. The Time Course of Similarity
Analogical Mapping:	1. Relational Similarity 2. One-to-One Correspondence 3. Parallel Connectivity 4. Systematicity 5. Flexibility 6. Scale

There are several benefits of simulation studies. First, simulation requires full specification of the psychological theory being simulated. Thus, Tuk-Tuk's assumptions about knowledge representation and processing are laid bare. Secondly, the validity of the psychological theory being simulated can be clearly assessed in terms of the degree to which simulation results match empirical findings. Thus, Tuk-Tuk's validity as a general theory of comparison is demonstrated by its ability to capture subjects' behavioral patterns. Thirdly, simulations can offer important insights into the psychological phenomena being simulated. Thus, a goal of the proposed simulations is to suggest new areas for empirical investigation. Potential directions for future research are discussed in Section 6.2.

5.3.1 Structured Representations

As described in Section 2.1.1, Markman and Gentner (2000) conducted a direct, concentrated study of the role of structure in comparisons that demonstrated that similarity processes utilize structured representations. Subjects were shown eight forced-choice triads and were asked to choose which of two target stimuli was most similar to a base stimuli.

This section presents simulations of each comparison.

Default parameters were used for all simulations. Figure 5.7 shows Tuk-Tuk's representations of the base and target stimuli for Triads 1 through 4b and Figure 5.8 shows Tuk-Tuk's representations of the base and target stimuli for Triads 5 and 6. For all triads, subjects preferred the target shown on the left over the target shown on the right. Figure 5.9 shows similarity trajectories generated by Tuk-Tuk for Triads 1 through 3 and Figure 5.10 shows similarity trajectories generated by Tuk-Tuk for Triads 4a through 6. For all triads, Tuk-Tuk generates higher similarity scores for the target that was preferred by subjects than for the target that was not preferred by subjects.

For Triad 1, Tuk-Tuk places the objects of the preferred target into correspondence with the objects of the base based on the objects' shapes, and these matching shapes increase Tuk-Tuk's similarity score. In contrast, the target that is not preferred has no features in common with the base, and Tuk-Tuk gives this comparison a similarity score of zero. These simulations verify that Tuk-Tuk finds stimuli with similar objects to be more similar than stimuli with dissimilar objects.

Triads 2a and 2b demonstrate that Tuk-Tuk finds stimuli with the same relation to be more similar than stimuli without the same relation, even when objects that play the same relational roles are different. In Triad 2a, the preferred target has the same relation as the base, but the shapes from the base are cross-mapped. The target that is not preferred has a different relation, but the shapes are the same. In both comparisons, Tuk-Tuk places objects into correspondence based on their shapes, and these matching shapes increase Tuk-Tuk's similarity score. In the preferred comparison, the matching relations also increase Tuk-Tuk's similarity score, even though the shapes are cross-mapped. Triad 2b differs from Triad 2a in that the shapes in the targets are different from those in the base. Again, in the preferred comparison, the matching relations increase Tuk-Tuk's similarity score.

An interesting question is which objects in the preferred target in Triad 2b correspond to which objects in the base. Based on the structural constraint of parallel connec-

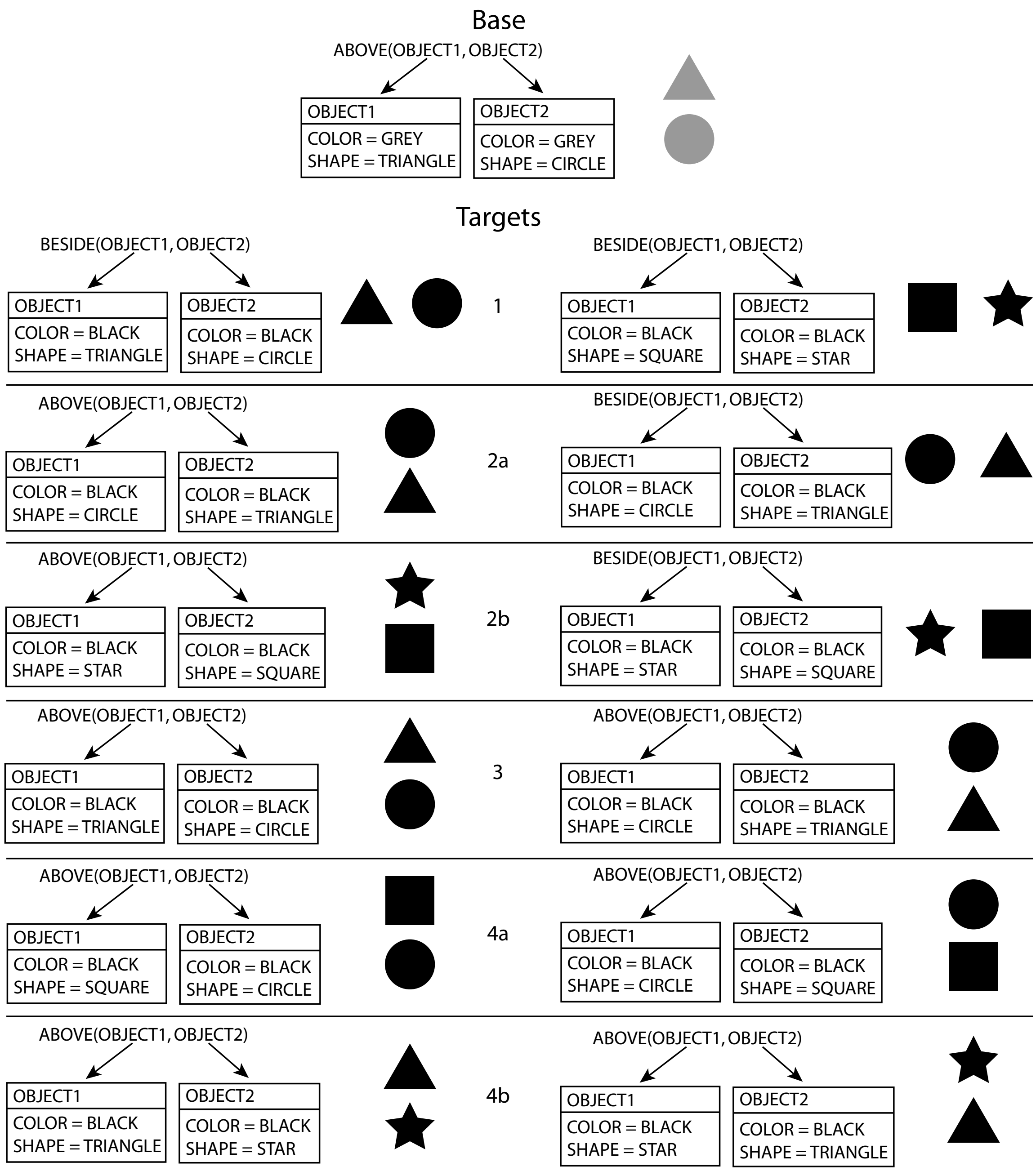

Figure 5.7: Tuk-Tuk's representations for simulating Triads 1 through 4b of Markman and Gentner's (2000) study.

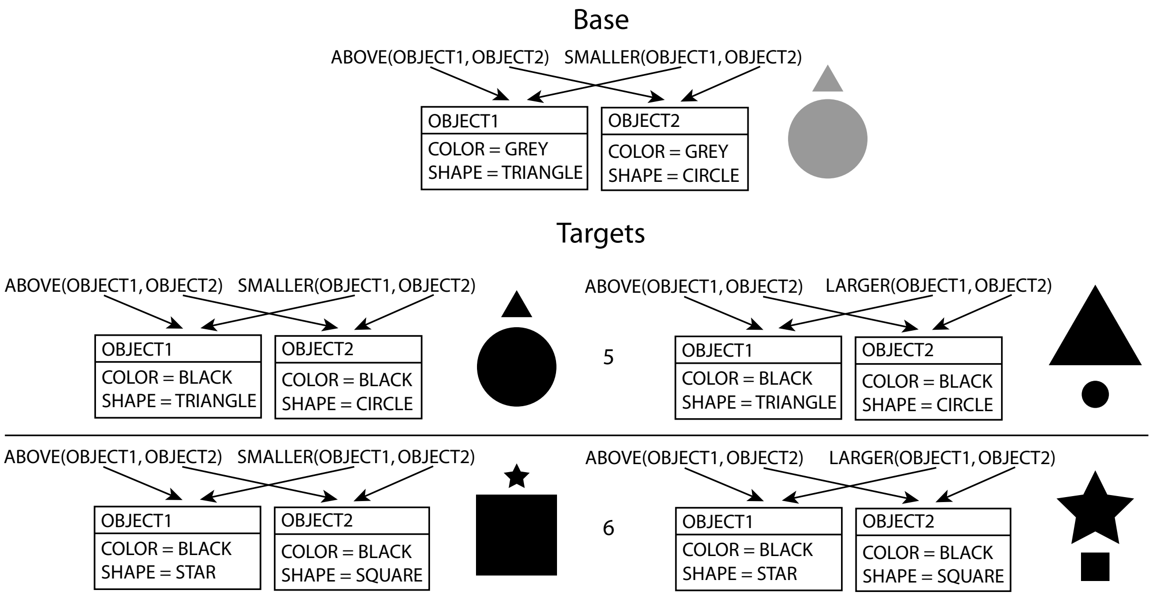

Figure 5.8: Tuk-Tuk's representations for simulating Triads 5 and 6 of Markman and Gentner's (2000) study.

tivity, Tuk-Tuk places the triangle into correspondence with the star and the circle into correspondence with the square because they fill the same relational roles.

Triad 3 shows that Tuk-Tuk finds stimuli with similar objects playing the same relational roles to be more similar than those with similar objects playing different relational roles. Early in processing, Tuk-Tuk gives both comparisons the same similarity score based on the matching shapes and relations, but over time Tuk-Tuk appreciates the comparison in which the correspondences between matching shapes and the correspondence between matching relations are structurally consistent. In the comparison that is not preferred, the shapes are cross-mapped, and this causes the activation associated with the matching relations to decrease, which decreases Tuk-Tuk's similarity score.

Triads 4a and 4b show that Tuk-Tuk finds stimuli with only one similar object playing the same relational role to be more similar than those having no similar objects playing the same relational role. Like Triad 3, Tuk-Tuk gives the preferred and not preferred com-

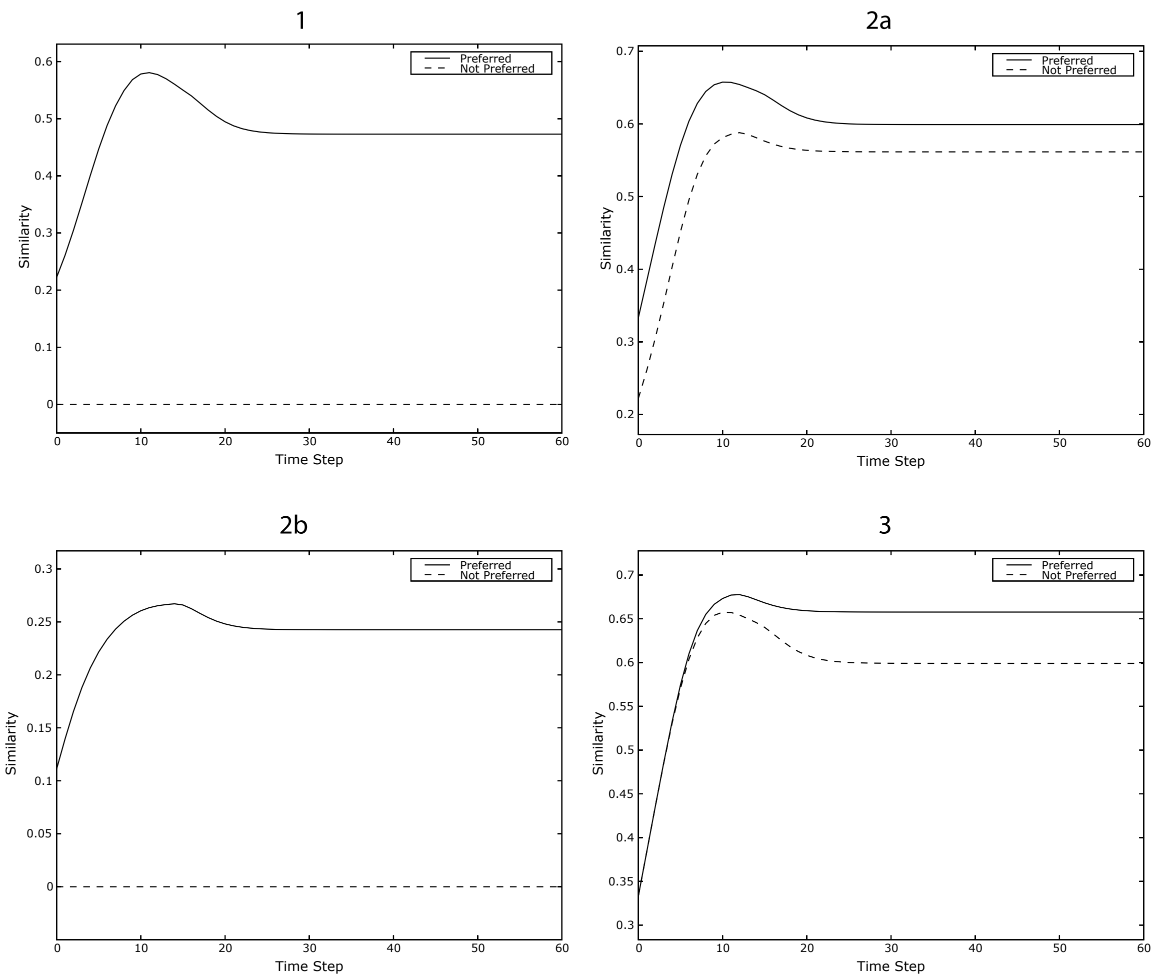

Figure 5.9: Similarity over time for Triads 1 through 3 of Markman and Gentner's (2000) study.

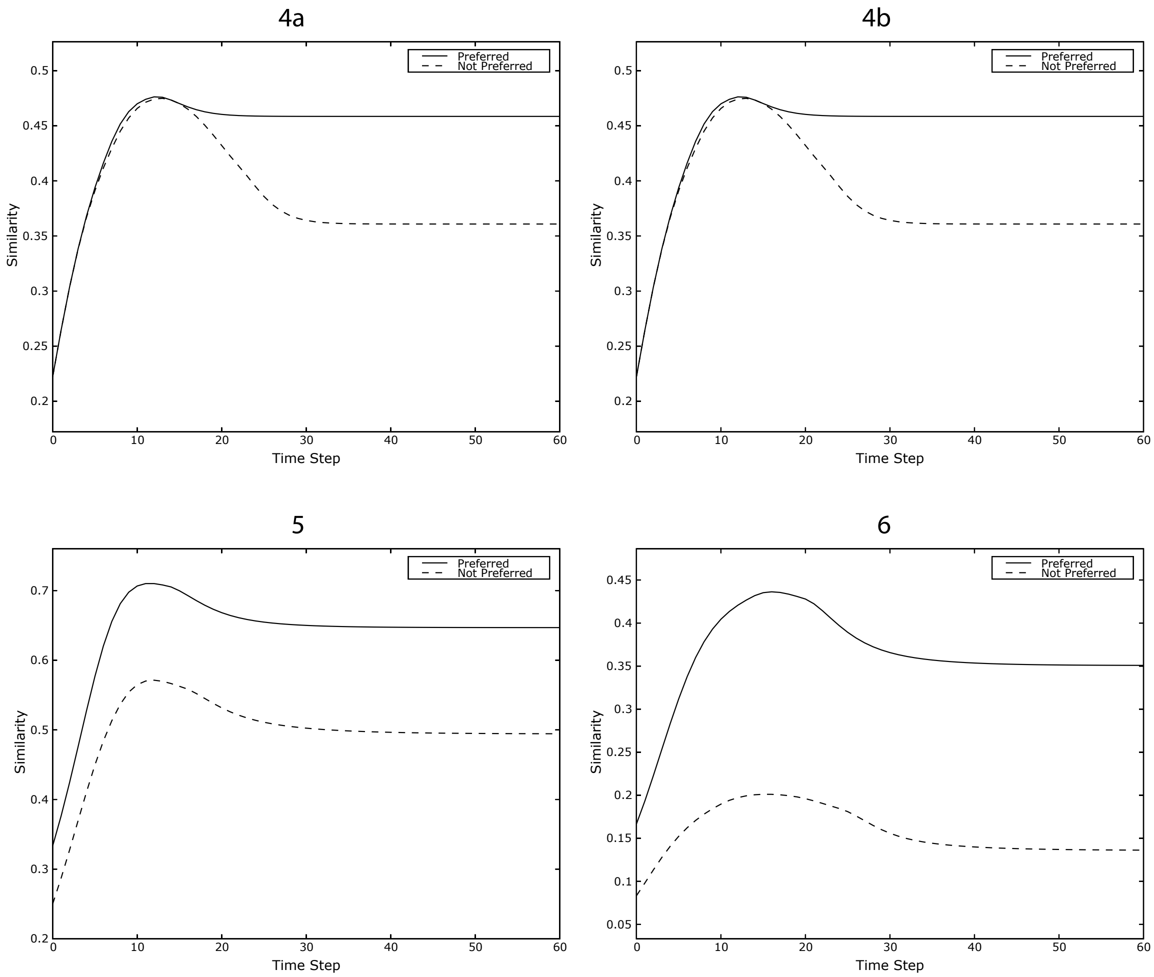

Figure 5.10: Similarity over time for Triads 4a through 6 of Markman and Gentner's (2000) study.

parisons the same similarity score early in processing, but over time Tuk-Tuk appreciates the comparison in which feature-to-feature and relation-to-relation correspondences are structurally consistent.

Triad 5 demonstrates that Tuk-Tuk prefers consistency across a number of relations in a scene. In the base, the triangle is above the circle, and the triangle is also smaller than the circle. The target that preserves both of these relational commonalities is preferred over the target that preserves only one of them. Triad 6 demonstrates that this preference holds even when the objects are different.

5.3.2 Matches in Place and Matches out of Place

Tuk-Tuk's ability to predict subjects' similarity judgments with respect to MIPs and MOPs is described in Section 5.1. Also, because Tuk-Tuk and SIAM produce identical similarity scores for comparisons that SIAM is capable of processing, Tuk-Tuk fully accounts for the patterns of similarity judgments demonstrated by Goldstone (1994) and described in Section 2.1.2.

An issue that cannot be addressed by SIAM is whether systematic relational structure shared by compared items amplifies the difference between MIPs and MOPs in influencing similarity. Representations for simulations that explore this issue are shown in Figures 5.11 and 5.12. Figure 5.11 (a) depicts a comparison between two scenarios, each involving three insects. The scenario on the left involves a big, black insect (i.e., INSECT1) and a medium-sized, red insect (i.e., INSECT2) eating a small, green insect (i.e., INSECT3). The scenario on the right involves a small, black insect (i.e., INSECT1) and a medium-sized, red insect (i.e., INSECT2) eating a small, green insect (i.e., INSECT3). Because INSECT3 on the left corresponds with INSECT3 on the right based on size, color, and being eaten, the feature SIZE = SMALL shared by these insects constitutes a MIP. In contrast, because INSECT3 on the left does not correspond with INSECT1 on the right, the feature SIZE = SMALL shared by these insects constitutes a MOP.

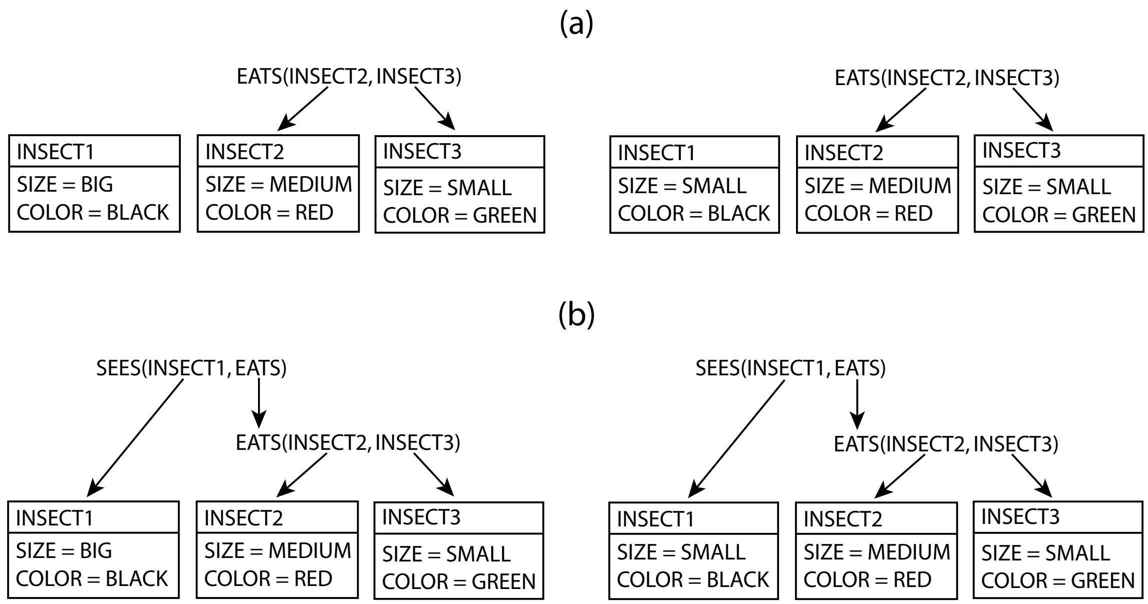

Figure 5.11: Tuk-Tuk's representations for simulating the influence of systematicity on MIPs and MOPs when corresponding entities share several features.

The scenarios in Figure 5.11 (b) differ from the scenarios in Figure 5.11 (a) in that INSECT1 sees INSECT2 eat INSECT3. The higher-order relation SEES adds to the systematicity of the relational structure shared by the scenarios. Figure 5.12 differs from Figure 5.11 in that corresponding insects are less similar in Figure 5.12 than in Figure 5.11.

Two sensible predictions are that (1) systematicity will amplify the difference between the MIP shared by INSECT3 and INSECT3 and the MOP shared by INSECT3 and INSECT1 as reflected by the associated node activations, and (2) the influence of systematicity will be greater when corresponding insects share few features. The intuition underlying the first prediction is that systematic comparisons will generate stronger correspondences, which will result in greater emphasis of corresponding matches over noncorresponding matches. The intuition underlying the second prediction is that the influence of systematicity will be greater in comparisons where correspondences are not already strong based on shared features.

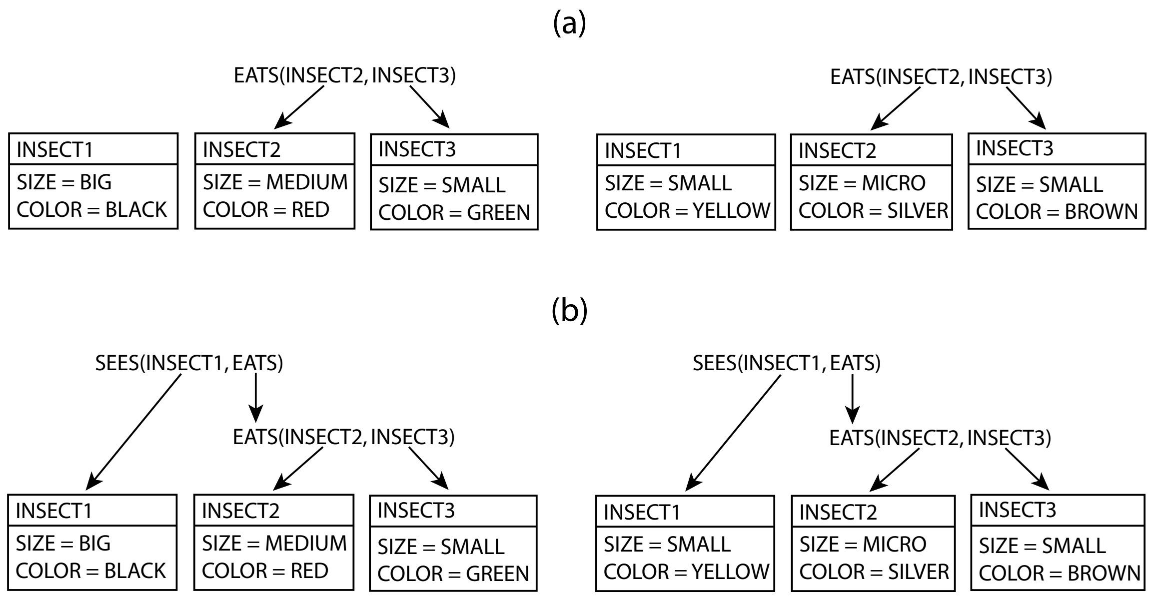

Figure 5.12: Tuk-Tuk's representations for simulating the influence of systematicity on MIPs and MOPs when corresponding entities share few features.

Figures 5.13 and 5.14 give simulation results for the comparisons depicted in Figures 5.11 and 5.12, respectively. Default parameters were used for all simulations. Consistent with previous findings, MIPs have a greater influence on similarity than MOPs. As predicted, the difference between the MIP shared by INSECT3 and INSECT3 and the MOP shared by INSECT3 and INSECT1 is greater for the systematic comparison than the nonsystematic comparison. That is, the activation of the MIP is higher in the systematic comparison than in the nonsystematic comparison and the activation of the MOP is lower in the systematic comparison than in the nonsystematic comparison. In addition, the influence of systematicity is greater when corresponding insects share few features. In particular, the activation of the MIP is substantially higher in the systematic comparison than in the nonsystematic comparison when corresponding insects share few features (see Figure 5.14). In contrast, the activation of the MIP is only slightly higher in the systematic comparison than in the nonsystematic comparison when corresponding insects share

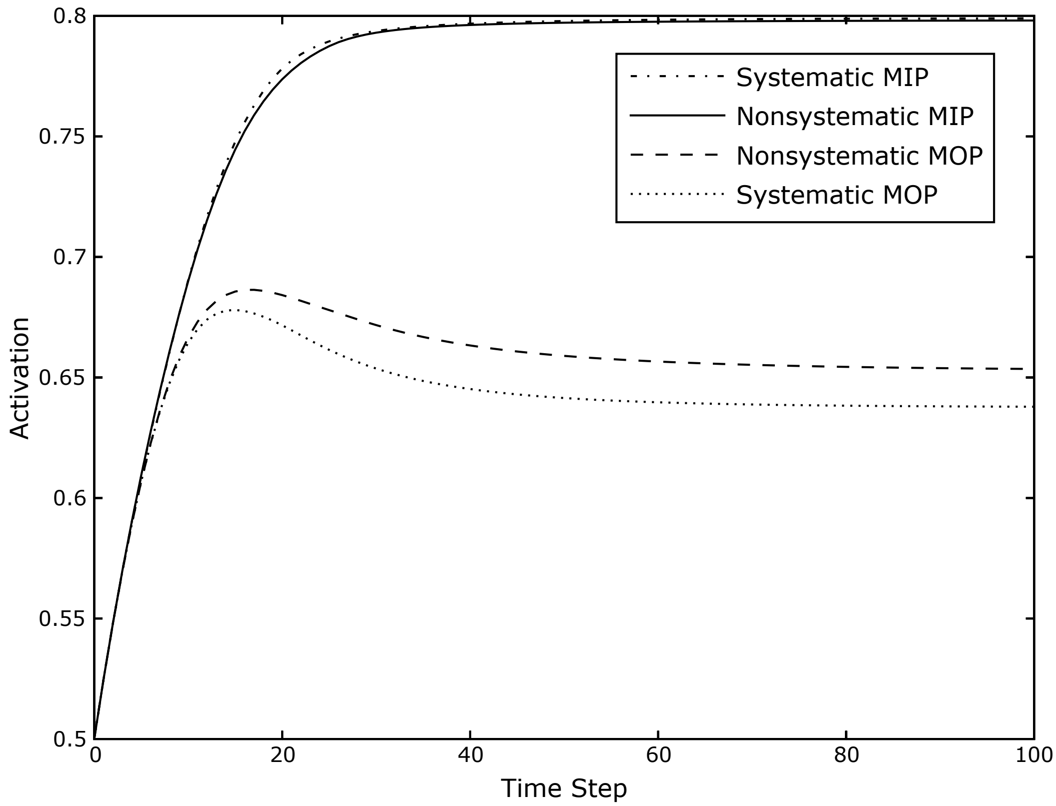

Figure 5.13: Activations over time when corresponding insects share several features.

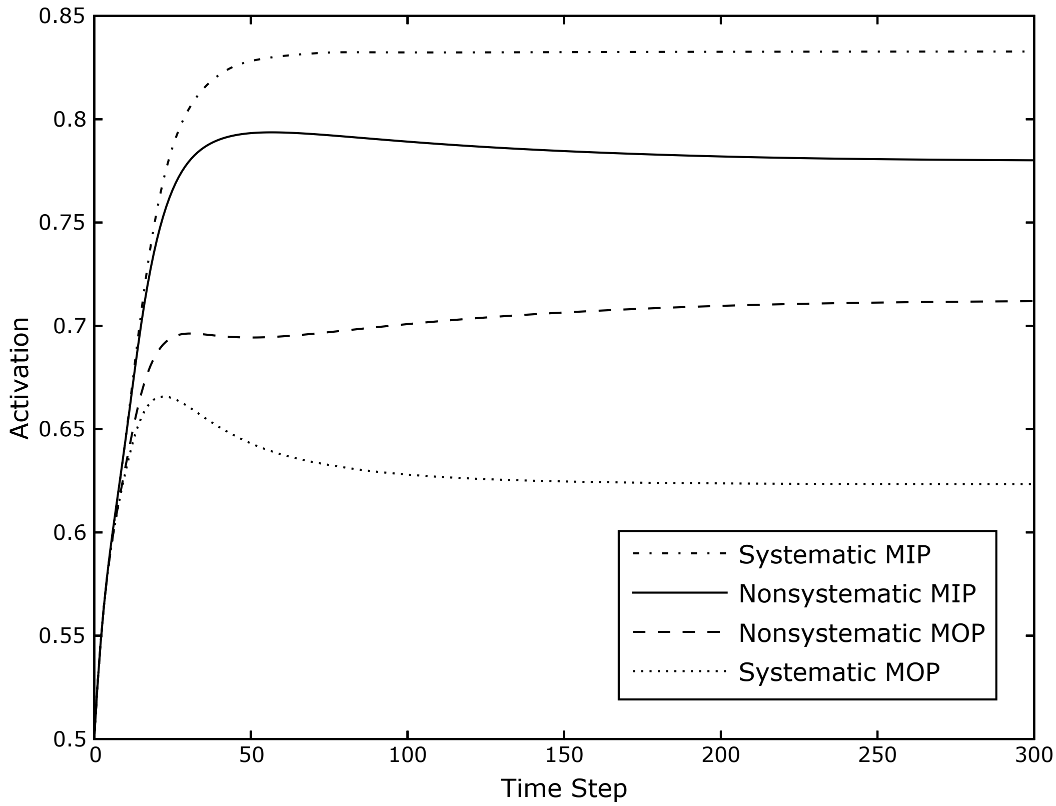

Figure 5.14: Activations over time when corresponding insects share few features.

several features (see Figure 5.13). One way to view this result is as a ceiling effect on correspondence strength. In other words, because the correspondence between INSECT3 and INSECT3 is already strong based on size, color, and being eaten (see Figure 5.11), adding to the systematicity of the relational structure shared by the scenarios can strengthen the correspondence only a little.

5.3.3 Alignable and Nonalignable Differences

This section describes two sets of simulations that verify Tuk-Tuk's ability to account for alignable versus nonalignable differences. The first set of simulations apply Tuk-Tuk to Markman and Gentner's (1996) study that demonstrates that variations in alignable differences affect similarity more than variations in nonalignable differences as described in Section 2.1.3. Abstract representations of the stimuli used in the study are shown in Figure 5.15. R denotes a relation, E1, E2, E3, and E4 denote different entities, and D = V1, D = V2, D = V3, and D = V4 denote different features that have the same dimension. Figure 5.15 (a) is compared separately with each of (b), (c), (d), and (e). In the comparison between (a) and (b), D = V2 versus D = V3 is an alignable difference. In the comparison between (a) and (c), D = V2 versus D = V4 is also an alignable difference, but the difference between D = V2 and D = V4 is greater than the difference between D = V2 and D = V3. This distinction is captured by a lower match value for D = V2 and D = V4 than for D = V2 and D = V3. In (d) and (e), the same two features (D = V3 and D = V4, respectively) are nonalignable differences constituting entities that do not correspond with any entity in (a). When subjects rated the similarity of all four comparisons of the base (a) with the targets (b), (c), (d), and (e), there was a greater difference in rated similarity between the two alignable difference pairs ((a) with (b) and (a) with (c)) than between the two nonalignable difference pairs ((a) with (d) and (a) with (e)).

Each comparison was simulated using default parameters except the match value for D = V2 and D = V3 was set to 0.5. Simulation results are shown in Figure 5.16. Like

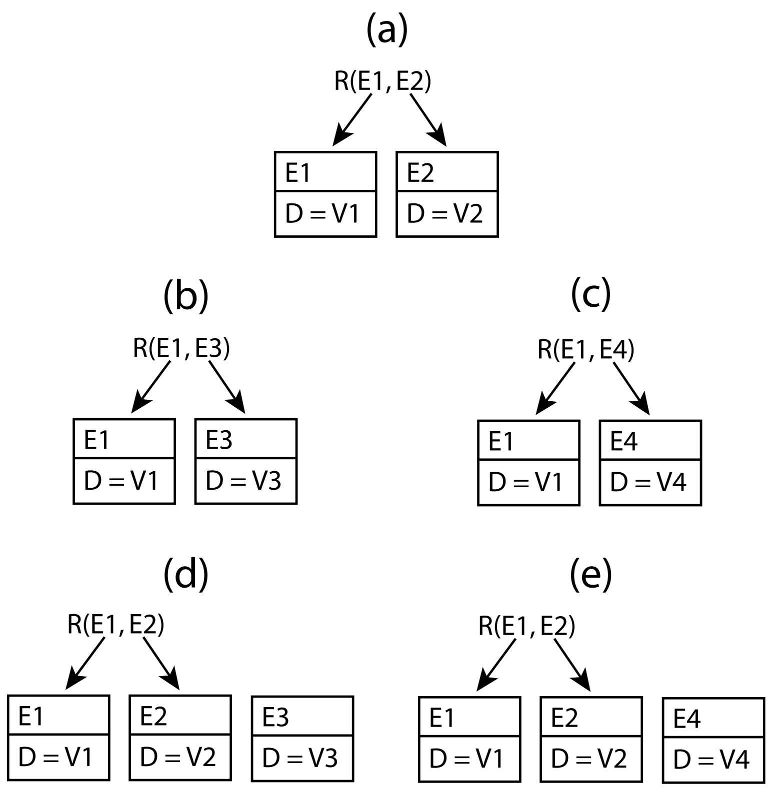

Figure 5.15: Tuk-Tuk's representations using variable match values to simulate Markman and Gentner's (1996) study demonstrating that variations in alignable differences affect similarity more than variations in nonalignable differences.

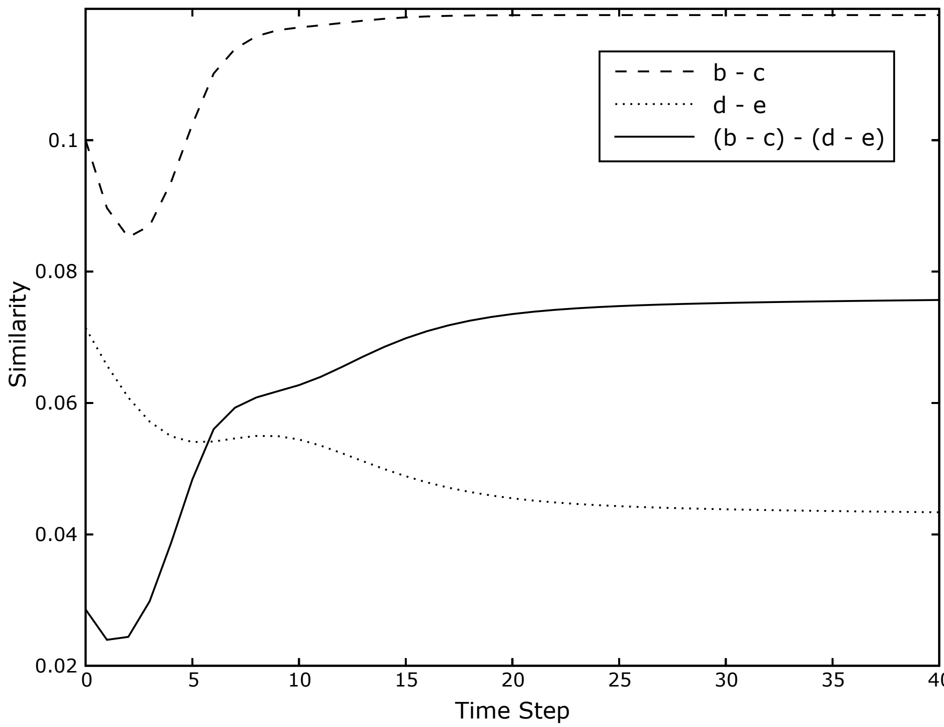

Figure 5.16: Simulation results using variable match values. The difference between the two alignable difference pairs is labeled b - c and the difference between the two nonalignable difference pairs is labeled d - e.

subjects, Tuk-Tuk generates a greater difference in similarity between the two alignable difference pairs ((a) with (b) and (a) with (c)) than between the two nonalignable difference pairs ((a) with (d) and (a) with (e)).

A potential problem with using variable match values to capture semantic similarity is that it assumes a similarity process that is external to Tuk-Tuk. This problem can be rectified by decomposing elements that are similar but not identical such that their similarity is expressed as partial identity. For example, a car, a truck, and a robot are all different, but a car is more similar to a truck than to a robot because a car shares more features with a truck than with a robot.

Figure 5.17 shows abstract representations of Markman and Gentner's (1996) stimuli

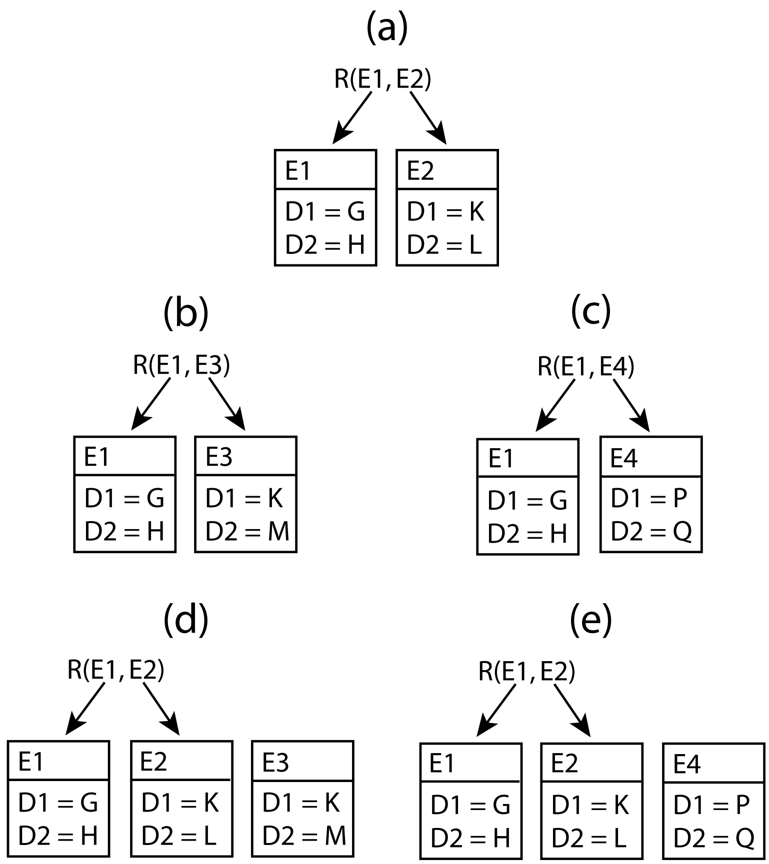

Figure 5.17: Tuk-Tuk's representations using decomposition to simulate Markman and Gentner's (1996) study.

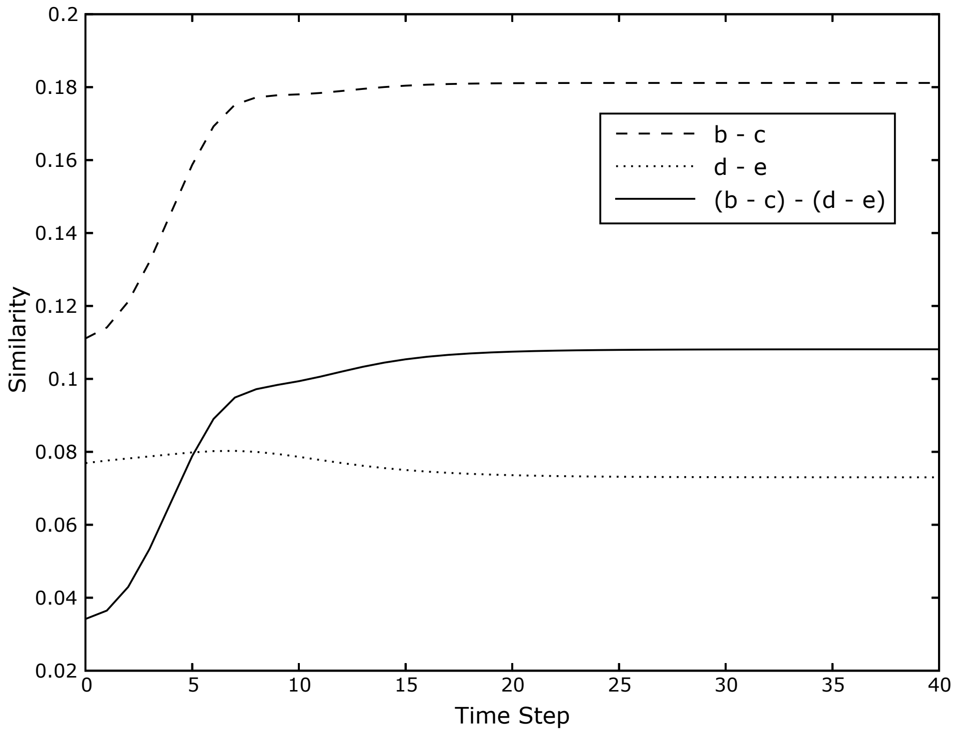

Figure 5.18: Simulation results using decomposition.

using decomposition to represent entities that are similar but not identical. R denotes a relation, E1, E2, E3, and E4 denote different entities, and D1 = G, D1 = K, D1 = P, D2 = H, D2 = L, D2 = M, and D2 = Q denote different features, where D1 and D2 are different dimensions. The difference between E2 and E4 is greater than the difference between E2 and E3 because E2 and E4 have two mismatching features whereas E2 and E3 have one matching feature and one mismatching feature.

Figure 5.18 gives simulation results using default parameters. As above, variations in alignable differences have a greater influence on similarity than variations in nonalignable differences.

The second set of simulations apply Tuk-Tuk to Markman and Gentner's (1997) study demonstrating that alignable differences are better memory probes than nonalignable differences as described in Section 2.1.3. Table 5.3 gives representations of the stimuli shown

Table 5.3: Tuk-Tuk's representations for simulating Markman and Gentner's (1997) study demonstrating that alignable differences are better memory probes than nonalignable differences.

Target	Base
PIG	BABY
TRACTOR	WALL
FARMER	MOTHER
HELICOPTER	
HAY	
SMILING(PIG)	SMILING(BABY)
MESSES(PIG, TRACTOR)	MESSES(BABY, WALL)
YELLS AT(FARMER, PIG)	YELLS AT(MOTHER, BABY)
CAUSES(MESSES, YELLS AT)	CAUSES(MESSES, YELLS AT)
BLOWS(HELICOPTER, HAY)	

in Figure 2.4. The pig and the baby constitute an alignable difference because they play similar roles of making a mess which causes someone to yell at them. They also share the attribute of smiling. In contrast, the helicopter is a nonalignable difference because it does not correspond with any entity in the base. Subjects were able to recall more about the scene when cued with an alignable difference (e.g., the pig) than when cued with a nonalignable difference (e.g., the helicopter). This suggests that people attend to corresponding information more than noncorresponding information when making comparisons.

Node activations in Tuk-Tuk's network can be interpreted in terms of attention during encoding into memory. A high node activation indicates that a large amount of attention is paid to the representational elements associated with that node. This interpretation is consistent with the idea that the influence of a particular correspondence on similarity increases with the activation of its associated node.

The comparison was simulated using default parameters. Maximal node activations associated with PIG and HELICOPTER at each time step are shown in Figure 5.19. Consistent with the interpretation that a larger amount of attention is paid to PIG than to HELICOPTER

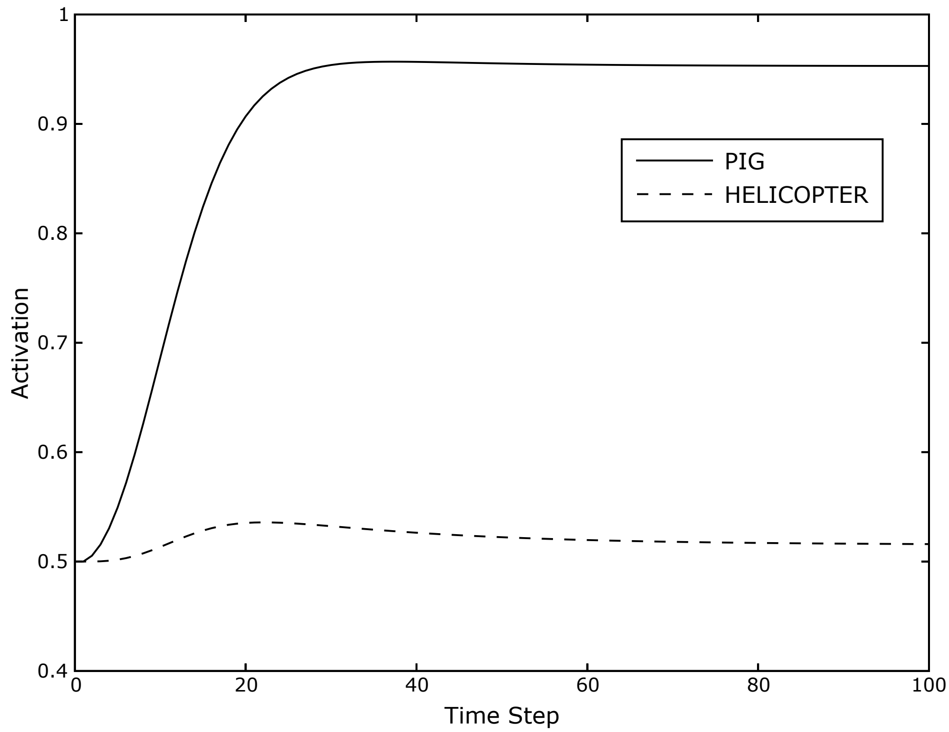

Figure 5.19: Maximal activations associated with PIG and HELICOPTER.

during encoding into memory, the maximal activation associated with PIG is higher than the maximal activation associated with HELICOPTER.

5.3.4 Nonmonotonicity

By virtue of reducing to SIAM for comparisons that SIAM is capable of processing, Tuk-Tuk fully accounts for the nonmonotonicities demonstrated by Goldstone (1996) and described in Section 2.1.4. These nonmonotonicities suggest that adding matching features to two items can decrease their similarity if the matching features promote correspondences that are inconsistent with correspondences between similar objects.

This phenomenon can be generalized to include analogies as well. Adding matching information to analogs that distracts from dominant correspondences should make an analogy more difficult to interpret. This section describes simulations of two versions of an analogy between water flow and heat flow (Falkenhainer et al., 1989; Gentner, 1983, 1989). Figure 5.20 shows illustrations and representations of the two analogs. The water flow analog involves water flowing from a large beaker filled with water through a pipe into a smaller vial because of a pressure difference. The heat flow analog involves a melting ice cube attached to a silver bar resting in a cup of hot coffee. The heat flows from the coffee through the bar to the ice cube because of the temperature difference.

Another version of the analogy was created by adding the distracting information that water and coffee are both liquids. This information is inconsistent with the dominant correspondences in suggesting that water should correspond with coffee.

Both versions of the analogy were simulated using default parameters. Simulation results are shown in Figure 5.21. The activation of each dominant correspondence is lower for the inconsistent version than for the basic version. Tuk-Tuk's correspondences are less certain for the inconsistent version than for the basic version even though the inconsistent version has additional matching information. For example, the correspondence between water and heat is less certain in the inconsistent version than in the basic version because the inconsistent information that water and coffee are both liquids excites the correspondence between water and coffee, which inhibits the correspondence between water and heat. Thus, Tuk-Tuk predicts a nonmonotonicity where adding matching information makes an analogy more difficult to interpret if the information is inconsistent with the dominant interpretation of the analogy.

5.3.5 Asymmetry

While Tuk-Tuk's similarity scores do not depend on the direction of comparison, Tuk-Tuk is compatible with the coherence imbalance hypothesis (Gentner & Bowdle, 1994; Bowdle & Gentner, 1997), which posits that asymmetries reflect differences in the informativeness of each direction of comparison (see Section 2.1.5). The coherence imbalance hypothesis

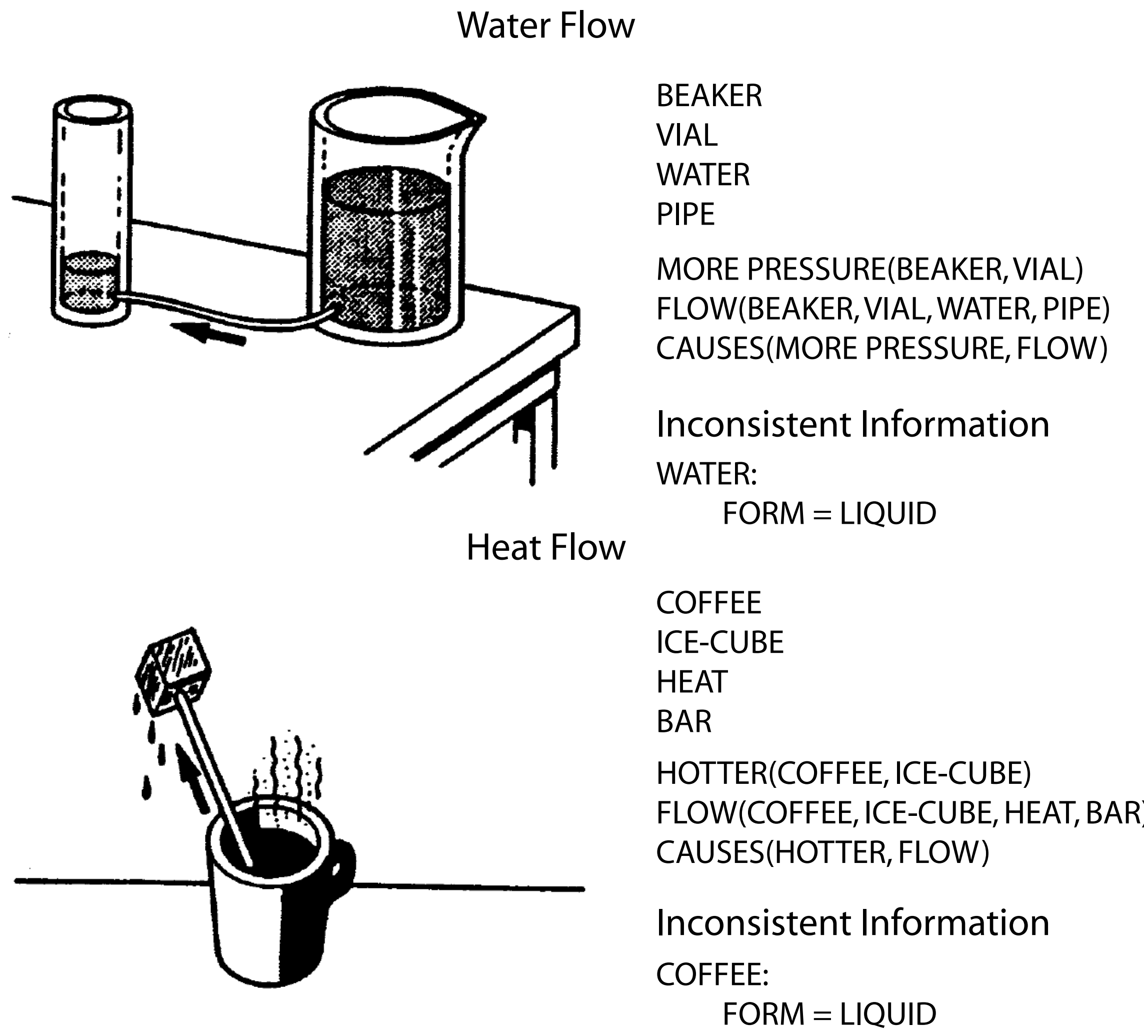

Figure 5.20: Tuk-Tuk's representations for simulating basic and inconsistent versions of an analogy between water flow and heat flow.

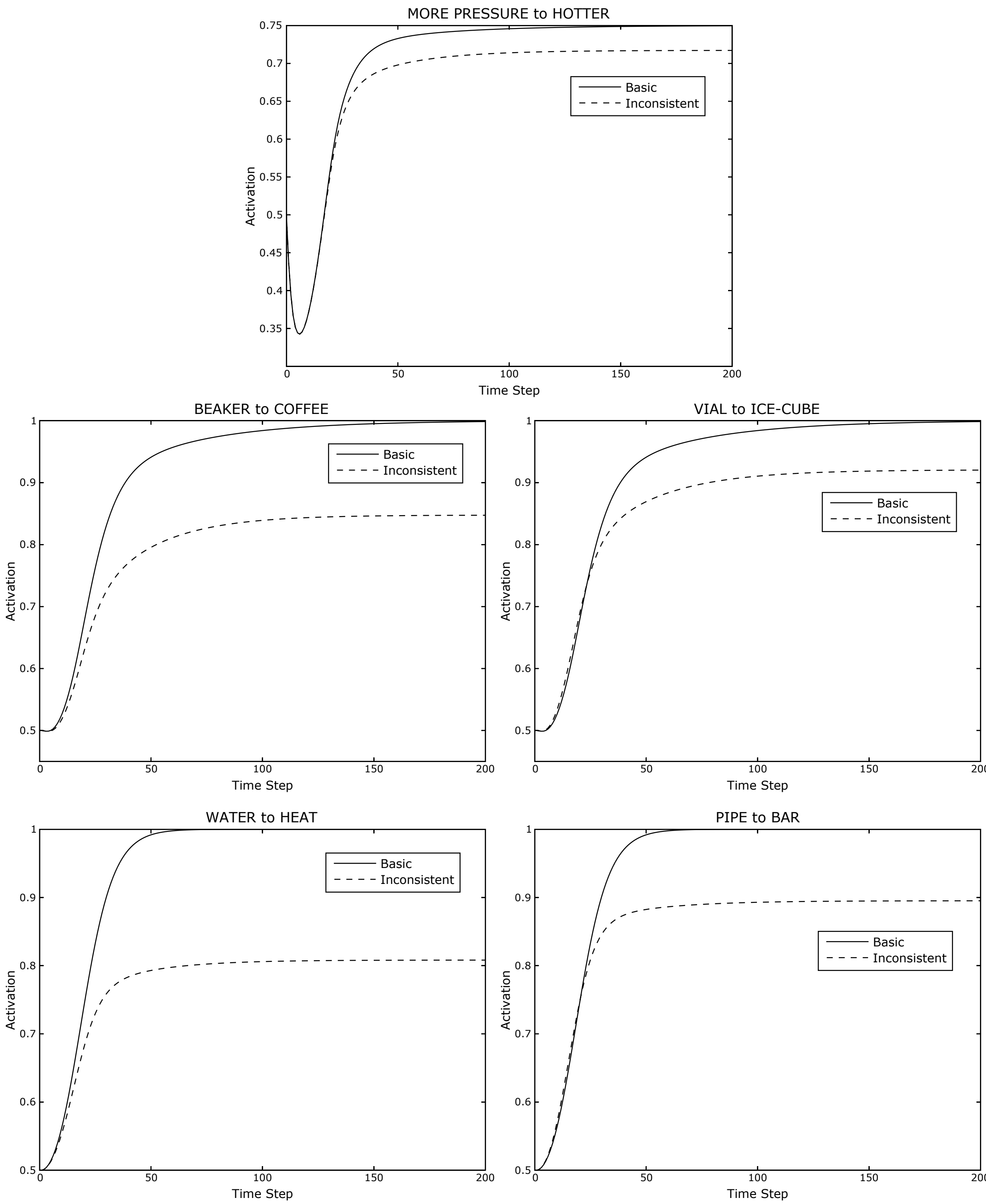

Figure 5.21: Activations over time for basic and inconsistent versions of an analogy between water flow and heat flow.

derives asymmetries from differences between the coherence of the target and the base. People prefer the direction of comparison where the base is more coherent than the target because this supports more inferences than does the opposite direction of comparison. The mappings Tuk-Tuk generates are suitable for the inference process of copying with substitution and generation (Holyoak et al., 1994). While Tuk-Tuk's similarity scores are symmetric, copying with substitution and generation is asymmetric, projecting inferences from the base to the target. Thus, while asymmetries do not emerge from Tuk-Tuk's processing, they do emerge from the process of copying with substitution and generation, which can take Tuk-Tuk's mappings as input.

5.3.6 The Time Course of Similarity

Because Tuk-Tuk and SIAM produce identical similarity scores for comparisons that SIAM is capable of processing, Tuk-Tuk fully accounts for the time course of similarity demonstrated by Goldstone and Medin (1994) and Goldstone (1996) and described in Section 2.1.6.

Simulations of Triads 3 through 4b of Markman and Gentner's (2000) study are consistent with the view that comparisons are driven by semantic commonalities early in processing and reflect structural constraints in time. For Triad 3 (see Figure 5.7), Tuk-Tuk finds stimuli with similar objects playing the same relational roles to be more similar than those with similar objects playing different relational roles. The time courses of these comparisons are of particular interest (see Figure 5.9). Early in processing, Tuk-Tuk gives both comparisons the same similarity score based on the matching shapes and relations, but over time Tuk-Tuk appreciates the comparison in which the correspondences between matching shapes and the correspondence between matching relations satisfy the structural constraint of parallel connectivity. In the comparison that is not preferred, the shapes are cross-mapped, and this causes the activation associated with the matching relations to decrease over time, which decreases Tuk-Tuk's similarity score. For Triads 4a and 4b, Tuk-Tuk finds stimuli with only one similar object playing the same relational role to be more similar than those having no similar objects playing the same relational role. Like Triad 3, Tuk-Tuk gives the preferred and not preferred comparisons the same similarity score early in processing, but over time Tuk-Tuk appreciates the comparison in which feature-to-feature and relation-to-relation correspondences are structurally consistent (see Figure 5.10).

Simulations of Markman and Gentner's (1996) study demonstrating that variations in alignable differences affect similarity more than variations in nonalignable differences are also consistent with the view that comparisons are driven by semantics early in processing and reflect structural constraints in time. Tuk-Tuk generates a greater difference in similarity between the alignable difference pairs ((a) with (b) and (a) with (c)) than between the two nonalignable difference pairs ((a) with (d) and (a) with (e)), and the magnitude of this effect increases with time (see Figures 5.16 and 5.18). Thus, the importance of being alignable versus nonalignable is realized over time.

Several simulations conducted with respect to other benchmarks have interesting time courses that cannot be addressed by SIAM. For example, in the simulations exploring the influence of systematicity on the difference between MIPs and MOPs, the difference between MIPs and MOPs is accelerated by systematicity, particularly when corresponding entities share few features (see Figures 5.13 and 5.14). Also, in the simulations of the analogy between water flow and heat flow, adding distracting information slows the trajectory of the dominant correspondences (see Figure 5.21). Additional time course predictions are discussed below with respect to the benchmarks of relational similarity and systematicity.

5.3.7 Relational Similarity

Tuk-Tuk is capable of processing comparisons throughout the similarity space described in Section 2.2.1 and shown in Figure 2.7. Tuk-Tuk's ability to account for analogical comparisons, which are based on common relational structure rather than object descriptions, is

Table 5.4: Tuk-Tuk's representations for a mere-appearance comparison.

Target	Base
JIM:	BILL:
GENDER = MALE	GENDER = MALE
BETTY:	CINDY:
GENDER = FEMALE	GENDER = FEMALE

demonstrated by the simulation of Rutherford's analogy between the atom and the solar system described in Section 5.2. In this analogy, the nucleus corresponds with the Sun and the electrons correspond with the planets because of shared relations (e.g., the electrons revolve around the nucleus as the planets revolve around the Sun). No features are shared by the electrons and the planets or the nucleus and the Sun.

Mere-appearance comparisons are based on object descriptions rather than relational structure. Table 5.4 shows representations for a simulation demonstrating that Tuk-Tuk can process comparisons involving only entities and the features that describe them. Using default parameters, Tuk-Tuk places the two males into correspondence and the two females into correspondence. Figure 5.22 shows activations over time for entity-to-entity and feature-to-feature correspondences. Tuk-Tuk first places GENDER = MALE into correspondence with GENDER = MALE and GENDER = FEMALE into correspondence with GENDER = FEMALE according to match values. These feature-to-feature correspondences then excite correspondences between the entities they describe, and Tuk-Tuk places JIM into correspondence with BILL and BETTY into correspondence with CINDY.

Literal similarity comparisons are based on both relational structure and object descriptions. Table 5.5 shows representations for simulations demonstrating that Tuk-Tuk can process comparisons involving both relations and features and that relations can override matching features in determining correspondences. There are two sensible interpretations of the comparison: a feature-based interpretation and a relational interpretation. In the feature-based interpretation, Jim corresponds with Bill and Betty corresponds with Cindy

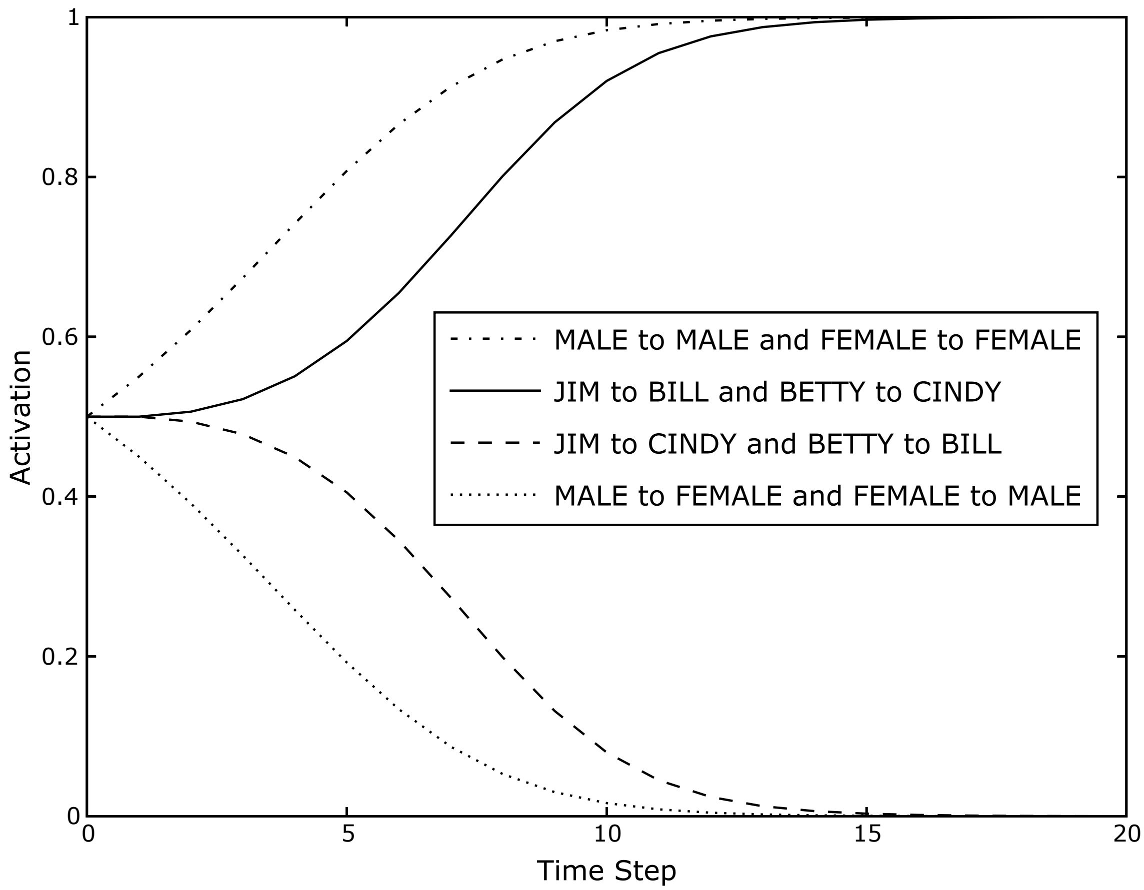

Figure 5.22: Activations over time for a mere-appearance comparison.

Table 5.5: Tuk-Tuk's representations for a literal similarity comparison.

Target	Base
Jim:	Bill:
gender = male	gender = male
Betty:	Cindy:
gender = female	gender = female
loves(Jim, Betty)	loves(Cindy, Bill)

based on their genders. In the relational interpretation, the correspondences reverse such that Jim corresponds with Cindy and Betty corresponds with Bill based on their respective relational roles (i.e., either loving the other or being loved by the other). In contrast to the feature-based interpretation, the matching loves relations govern the mapping.

Tuk-Tuk generates either the feature-based or the relational interpretation depending on the parameters relation-to-entity-wt and entity-to-feature-wt, which together serve as a proxy for relational focus because they govern the influence of relation-to-relation correspondences in determining structurally consistent mappings. Using default values for all other parameters, Tuk-Tuk's behavior changes abruptly between relation-to-entity-wt = entity-to-feature-wt = 2.1 and relation-to-entity-wt = entity-to-feature-wt = 2.2. With relation-to-entity-wt = entity-to-feature-wt $\leq$ 2.1 (i.e., low relational focus), Tuk-Tuk generates the feature-based interpretation. With relation-to-entity-wt = entity-to-feature-wt $\geq$ 2.2 (i.e., high relational focus), Tuk-Tuk generates the relational interpretation. Markman and Gentner (1993b) and Keane, Hackett, and Davenport (2001) suggest that the current task and context as well as relative frequency of relation versus feature matches in a stimulus pair and in a set of stimulus pairs can be factors that influence relational focus.

Activations over time for the feature-based interpretation are shown in Figure 5.23. Early in processing, Tuk-Tuk places the matching features into correspondence (i.e., gen-

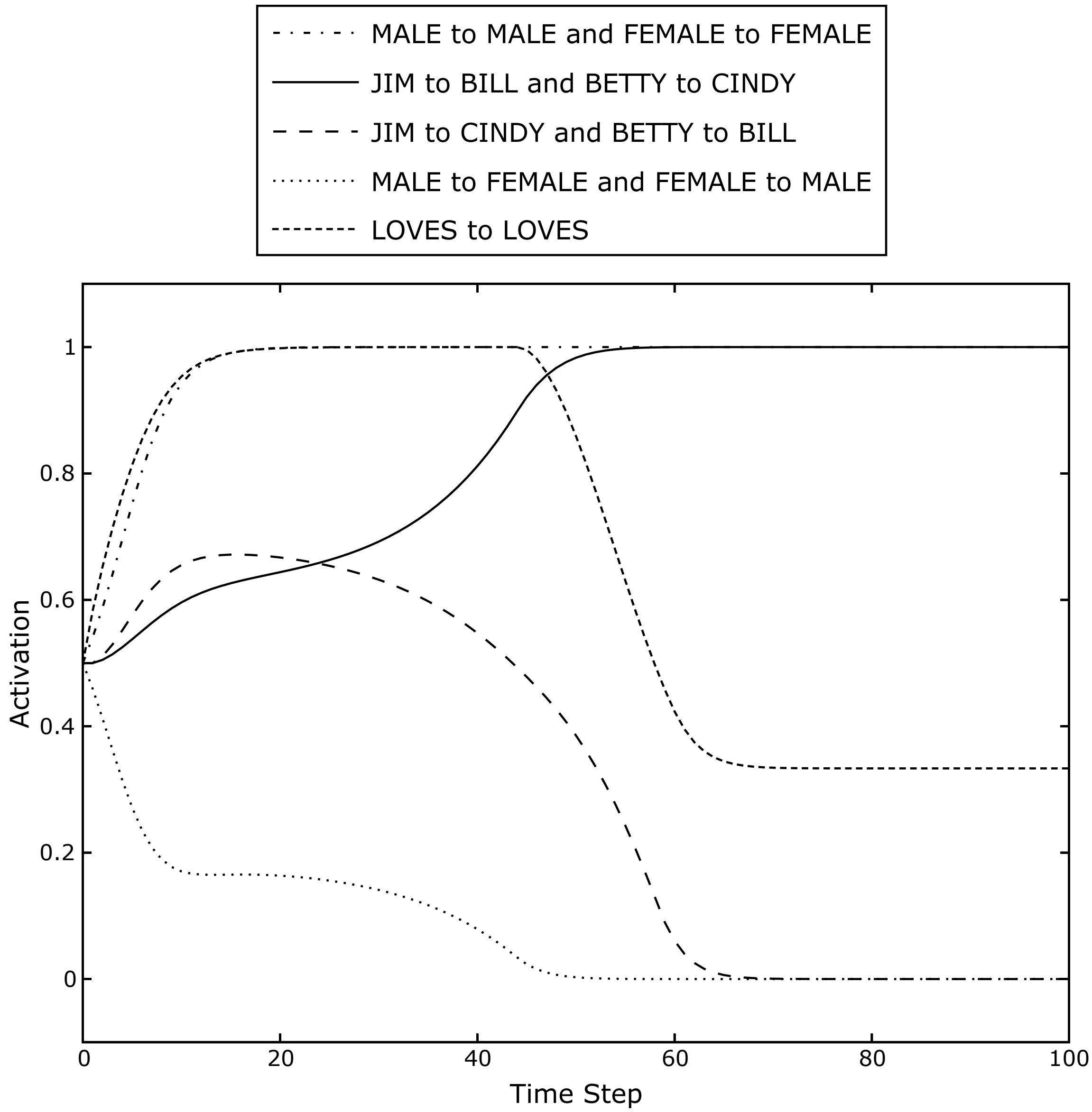

Figure 5.23: Activations over time for the feature-based interpretation.

der = male corresponds with gender = male and gender = female corresponds with gender = female) and the matching relations into correspondence (i.e., loves corresponds with loves) even though these correspondences are structurally inconsistent. Over time, the matching features bring their associated entities into correspondence (i.e., Jim corresponds with Bill and Betty corresponds with Cindy) and these entity-to-entity correspondences inhibit the inconsistent entity-to-entity correspondences such that Jim does not correspond with Cindy and Betty does not correspond with Bill. As a result, the correspondence between loves and loves weakens and the relational match is not included in Tuk-Tuk's final interpretation.

Activations over time for the relational interpretation are shown in Figure 5.24. As in the feature-based interpretation, Tuk-Tuk places the matching features into correspondence and the matching relations into correspondence early in processing even though these correspondences are structurally inconsistent. Over time, the matching relations bring their arguments into correspondence (i.e., Jim corresponds with Cindy and Betty corresponds with Bill) and these entity-to-entity correspondences inhibit the inconsistent entity-to-entity correspondences such that Jim does not correspond with Bill and Betty does not correspond with Cindy. In turn, the corresponding entities bring their associated features into correspondence such that the respective genders map consistently (i.e., Jim's feature gender = male corresponds with Cindy's feature gender = female and Betty's feature gender = female corresponds with Bill's feature gender = male). At the same time, the entities that do not correspond bring their associated features out of correspondence (i.e., Jim's feature gender = male does not correspond with Bill's feature gender = male and Betty's feature gender = female does not correspond with Cindy's feature gender = female). In addition, the dominant feature-to-feature correspondences inhibit the inconsistent correspondences between the matching features. Thus, for both the feature-based and relational interpretations, Tuk-Tuk arrives at a structurally consistent mapping over time as emerging entity correspondences override inconsistent re-

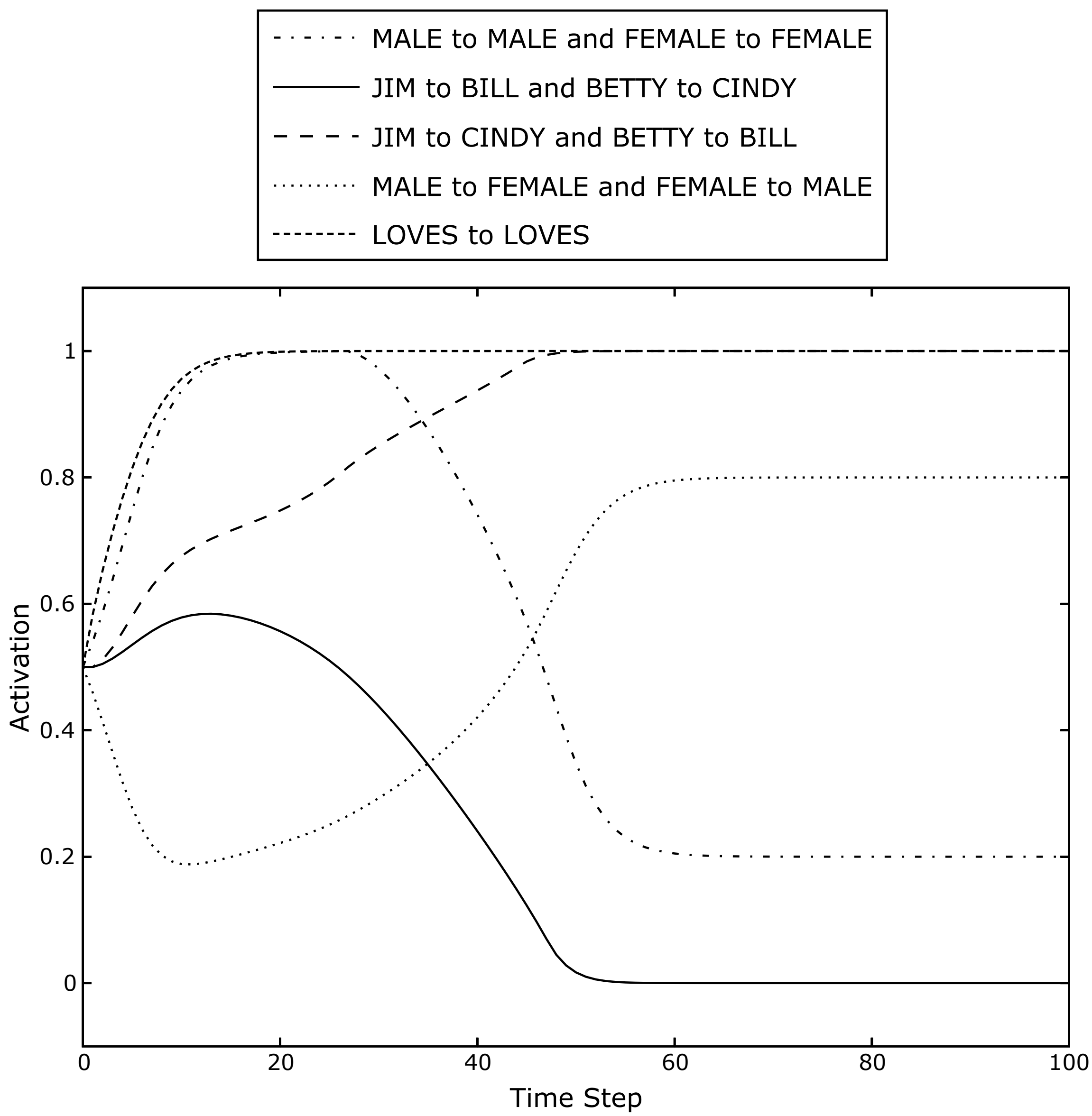

Figure 5.24: Activations over time for the relational interpretation.

lation or feature correspondences. This suggests that inconsistent correspondences should abound when comparisons are rushed.

5.3.8 One-to-One Correspondence

Patterns of similarity judgments (see Section 5.1) and correspondences reported by subjects (see Section 2.2.2) suggest that one-to-one correspondence is a soft constraint on the comparison process, yet subjects' inferences do not exhibit inconsistencies that follow from violations of one-to-one correspondence and copying with substitution and generation (see Section 2.2.2).

Tuk-Tuk applies a soft one-to-one constraint on correspondences via inhibitory connections between nodes that taken together place an element in one representation into correspondence with two elements in the other representation. This soft constraint allows Tuk-Tuk to predict patterns of similarity judgments that are inconsistent with the stricter one-to-one constraints of SME and CAB. However, Tuk-Tuk applies a strict one-to-one constraint when generating a mapping for the purpose of copying with substitution and generation. Thus, Tuk-Tuk posits that one-to-one correspondence is a soft constraint on people's mappings, but people strictly adhere to one-to-one correspondence prior to generating inferences. This differs from SME, which posits that reported correspondences violate one-to-one correspondence because people integrate correspondences across multiple global interpretations, each of which is strictly one-to-one.

5.3.9 Parallel Connectivity

Parallel connectivity is central to Tuk-Tuk's processing. Tuk-Tuk implements parallel connectivity such that correspondences between predicates are excited by correspondences between their arguments and vice versa. Likewise, correspondences between features are excited by correspondences between the entities they describe and vice versa. Every simulation described here is influenced by parallel connectivity. For example, in the simulation of Rutherford's analogy between the atom and the solar system described in Section 5.2, the nucleus corresponds with the Sun and the electrons correspond with the planets because the electrons revolve around the nucleus as the planets revolve around the Sun. That is, parallel connectivity allows that NUCLEUS and SUN correspond because both are the first argument to their respective REVOLVES AROUND relation, and that ELECTRONS and PLANETS correspond because both are the second argument to their respective REVOLVES AROUND relation (see Table 5.1).

5.3.10 Systematicity

The simulations described above with respect to MIPs and MOPs also bear upon systematicity. These simulations suggest that in a systematic comparison, MIPs should increase similarity more and MOPs should increase similarity less than in a nonsystematic comparison. In addition, the time-course of the difference between MIPs and MOPs should be accelerated by systematicity.

Higher-order causal information can be helpful in interpreting analogies (Holyoak & Koh, 1987; Gentner & Toupin, 1986). To explore the effects of causal structure on Tuk-Tuk's processing, an additional version of the analogy between water flow and heat flow described above was simulated. This version involves removing the causal relations from the basic version of the analogy shown in Fig 5.20. Removing these relations decreases the systematicity of the relational structure shared by the analogs.

Both versions of the analogy were simulated using default parameters. Simulation results are shown in Figure 5.25. The absence of the causal relations lowers the activations of the dominant correspondences such that Tuk-Tuk's correspondences are less certain for the less systematic version than for the basic version. In addition, Tuk-Tuk takes longer to determine correspondences for the less systematic version than for the basic version. For example, Tuk-Tuk takes much longer to appreciate the correspondence between MORE PRESSURE and HOTTER without the causal relations because the roles of MORE PRESSURE

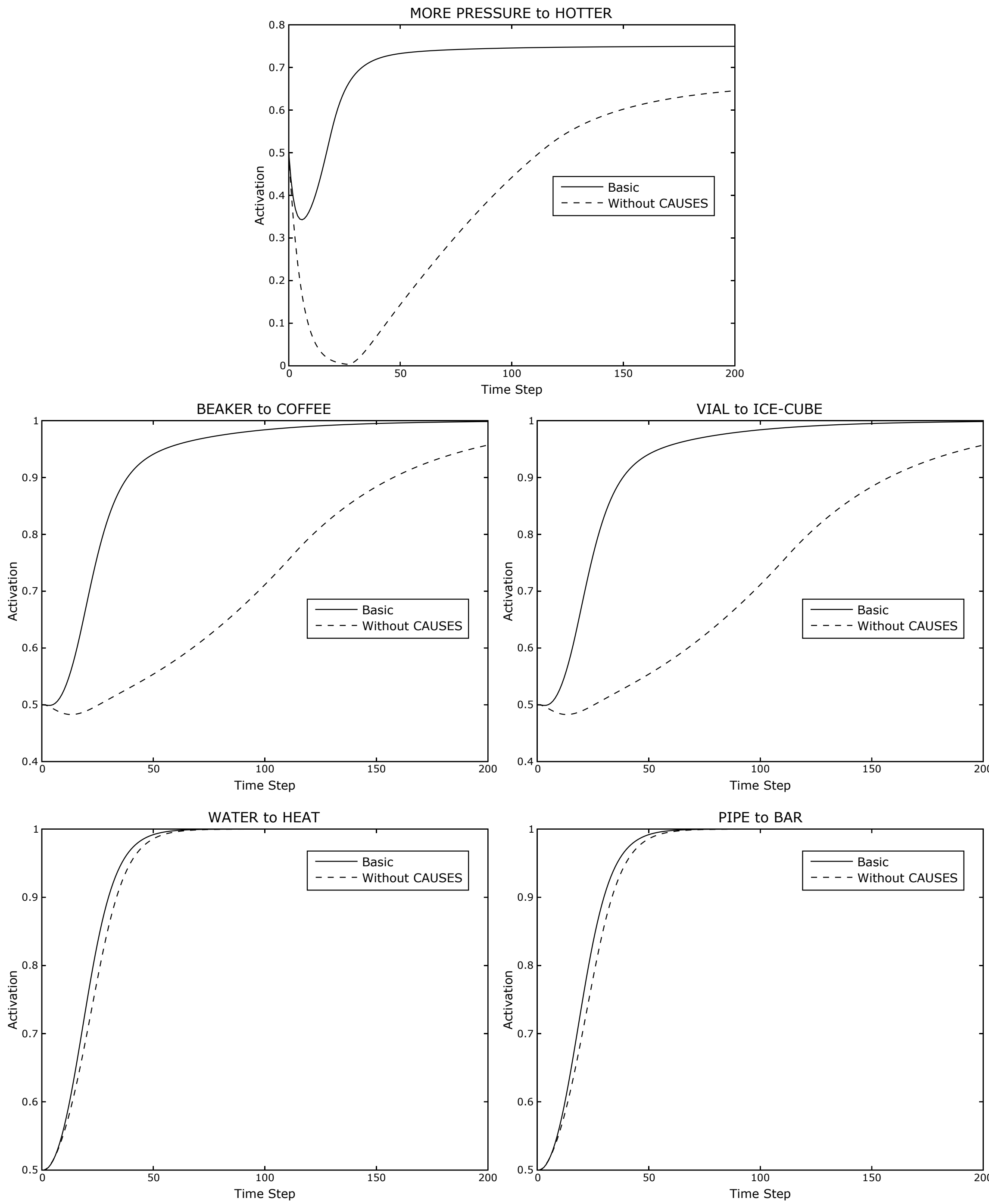

Figure 5.25: Activations over time for the basic version of the analogy between water flow and heat flow and for the version with the causal relation removed.

Table 5.6: Tuk-Tuk's representations for a cross-mapping with packed representations.

Target	Base
CAR	TRUCK
BOAT	CAR
TOWS(CAR, BOAT)	TOWS(TRUCK, CAR)

and HOTTER as causal antecedents are important factors in determining their correspondence. Thus, Tuk-Tuk predicts that decreasing the systematicity of an analogy will make the analogy more difficult to interpret such that correspondences are less certain and take longer to determine.

5.3.11 Flexibility

In cross-mappings, similar entities play different roles in matching relational structure (see Section 2.2.5). For example, Markman and Gentner (1993b) and Markman (1996) asked subjects to form correspondences between two pictures, one which depicted a car towing a boat and another which depicted a truck towing a car. People generally prefer the relationally consistent interpretation (e.g., the car in the first picture corresponding to the truck in the second picture), but can also appreciate the alternative mapping that places the two cars into correspondence.

Using default parameters, Tuk-Tuk generates either the relation-driven or the feature-driven mapping depending on differences in knowledge representation. When the entity encodings are not rich (see Table 5.6), Tuk-Tuk places entities into correspondence based on relational roles, but when the features of the entities are stressed (see Table 5.7), Tuk-Tuk generates the feature-driven mapping. This interpretation of cross-mapping is consistent with Stilwell and Markman's (2001) work on packing and unpacking of mental representations—when entities are construed as symbols (i.e., features are not stressed), relation-driven mappings abound. These simulations capture the benchmarks of interactive

Table 5.7: Tuk-Tuk's representations for a cross-mapping with unpacked representations.

Target	Base
CAR:	TRUCK:
COLOR = RED	COLOR = BLACK
SIZE = SMALL	SIZE = LARGE
MAKE = FORD	MAKE = CHEVY
BOAT:	CAR:
COLOR = BLUE	COLOR = RED
SIZE = MEDIUM	SIZE = SMALL
MAKE = NAUTIQUE	MAKE = FORD
TOWS(CAR, BOAT)	TOWS(TRUCK, CAR)

interpretation, multiple interpretations, and cross-mapping.

Activations over time for the relation-driven mapping are shown in Figure 5.26. The matching TOWS relations are placed into correspondence and over time bring their arguments into correspondence such that CAR corresponds to TRUCK and BOAT corresponds to CAR. In turn, these entity-to-entity correspondences inhibit the inconsistent entity-to-entity correspondences such that CAR does not correspond to CAR and BOAT does not correspond to TRUCK.

Activations over time for the feature-driven mapping are shown in Figure 5.27. Early in processing, Tuk-Tuk places the matching features into correspondence (e.g., COLOR = RED corresponds to COLOR = RED) and the matching TOWS relations into correspondence even though these correspondences are structurally inconsistent. Over time, the matching features bring CAR into correspondence with CAR, and this correspondence excites the consistent correspondence between BOAT and TRUCK and inhibits the inconsistent correspondences between CAR and TRUCK and between BOAT and CAR. At the same time, the mismatching features increase Tuk-Tuk's certainty that CAR does not correspond to TRUCK and BOAT does not correspond to CAR. Finally, Tuk-Tuk arrives at a structurally consistent mapping as the correspondence between BOAT and TRUCK brings the associated features

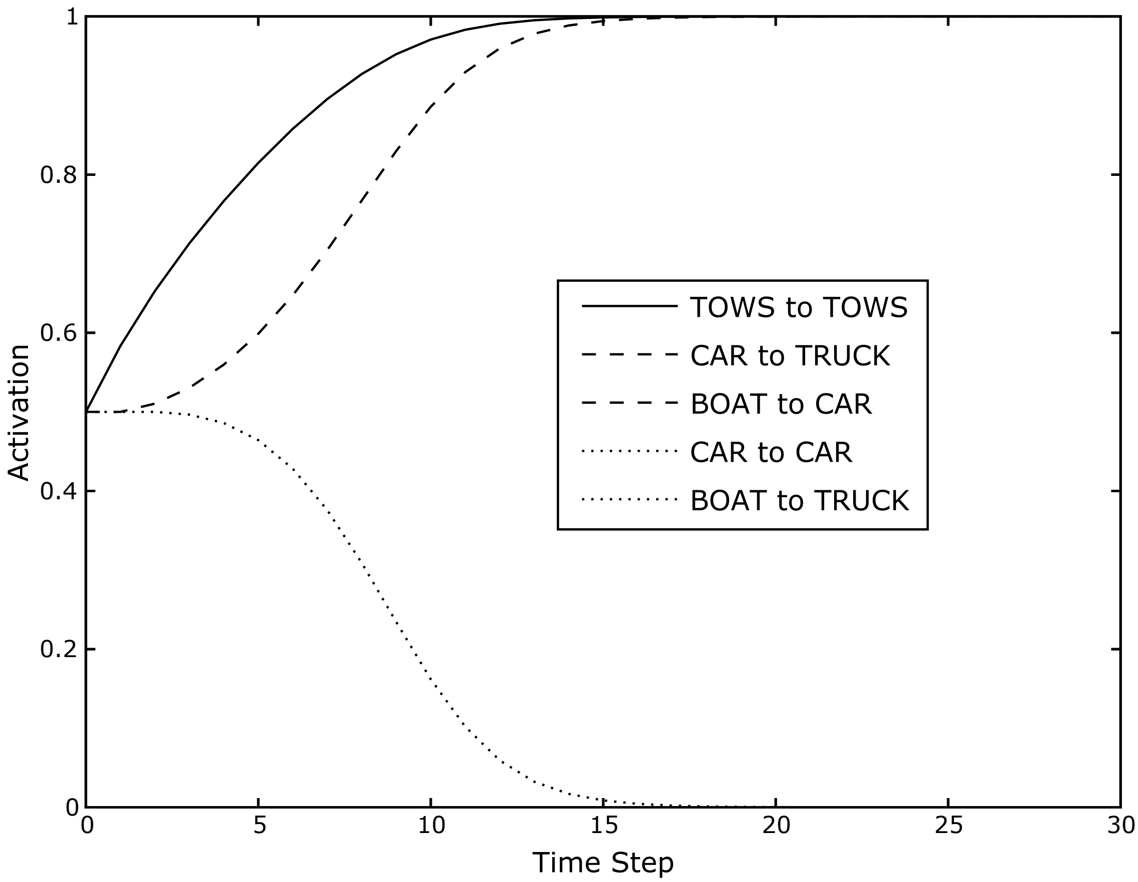

Figure 5.26: Activations over time for the relation-driven mapping.

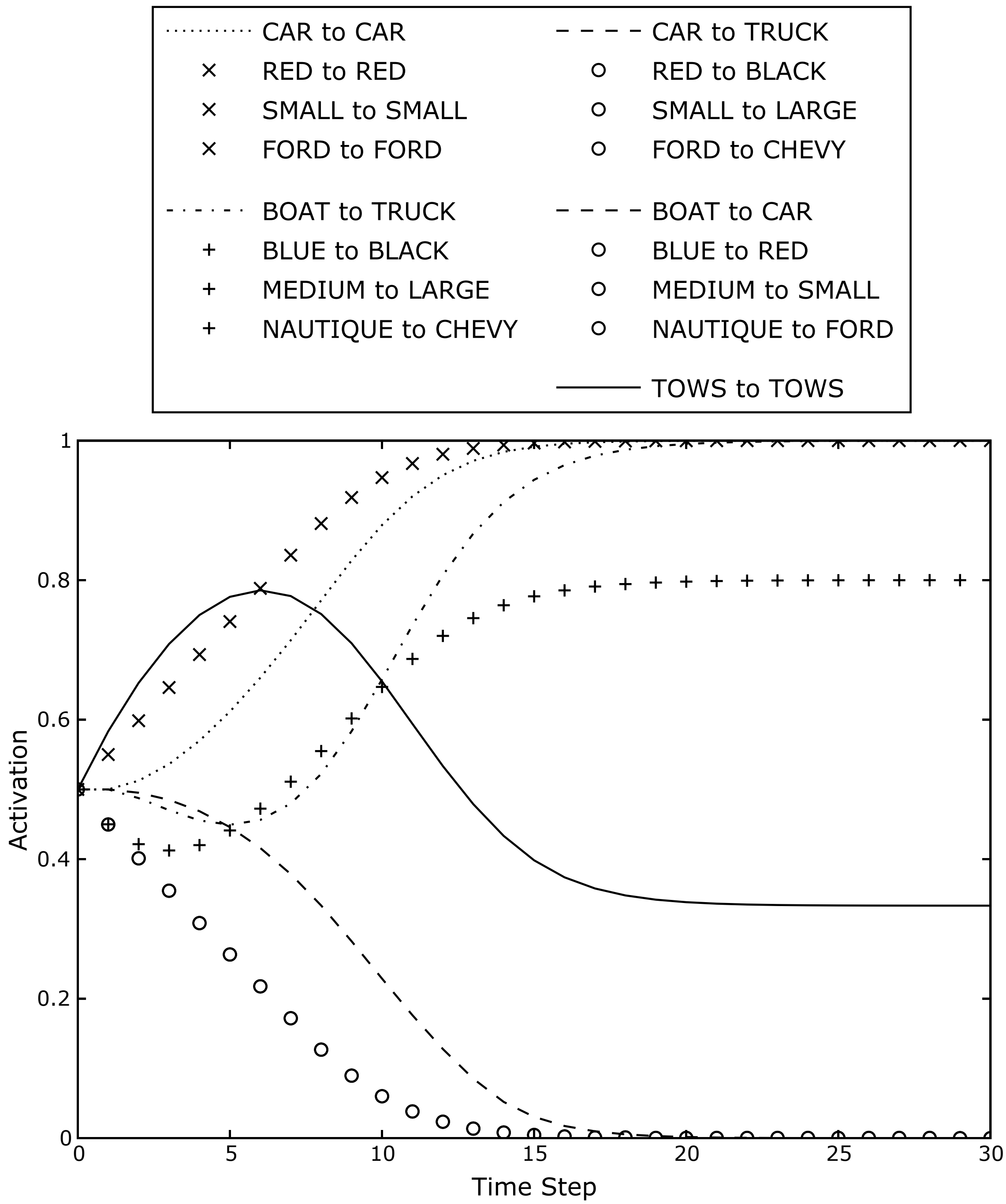

Figure 5.27: Activations over time for the feature-driven mapping.

into correspondence (e.g., COLOR = BLUE corresponds to COLOR = BLACK) and the non-corresponding arguments of the TOWS relations bring the relations out of correspondence.

5.3.12 Scale

One challenge for models of analogy is scaling to full human performance. In this section, Tuk-Tuk's ability to scale to analogies involving large representations is verified by simulating the "Karla the hawk" analogy used in studies conducted by Gentner and Landers (1985) and Gentner et al. (1993). This analogy has previously been simulated by SME (Falkenhainer et al., 1989). The analogy compares a story about a hunter and a hawk with a story about two countries. Each story involves several entities (e.g., Karla, a hunter, a cross-bow, feathers) and many interconnected relations (e.g., Karla giving the hunter her feathers causes the hunter to be grateful, which causes the hunter to promise not to attack Karla). Representations of these stories are shown in Table 5.8.

With relation-to-relation-inhibit-wt ≥ 1.6, feature-to-feature-inhibit-wt ≥ 1.1, and default values for all other parameters, Tuk-Tuk generates the correct mapping. The parameter relation-to-relation-inhibit-wt controls inhibition between inconsistent relation-to-relation correspondences that taken together place a relation in one representation into correspondence with two relations in the other representation. The parameter feature-to-feature-inhibit-wt controls inhibition between inconsistent feature-to-feature correspondences that taken together place a feature in one representation into correspondence with two features in the other representation. Consistent with the parameter study conducted with respect to Rutherford's analogy between the atom and the solar system (see Section 5.2), Tuk-Tuk's sensitivity to these parameters underscores the importance of the structural constraint of one-to-one correspondence. In addition to being a prerequisite for consistent inferences, the constraint of one-to-one correspondence establishes competition between correspondences that proves critical in generating certainty that elements correspond or do not correspond. When the influence of one-to-one correspondence is weak,

Table 5.8: Tuk-Tuk's representations for simulating the "Karla the hawk" analogy.

Target	Base
ZERDIA:	KARLA:
TYPE = COUNTRY	TYPE = HAWK
GAGRACH:	HUNTER:
TYPE = COUNTRY	TYPE = PERSON
DEMEANOR = WARLIKE	DEMEANOR = WARLIKE
MISSILES:	CROSS-BOW:
TYPE = WEAPON	TYPE = WEAPON
SUPERCOMPUTER	FEATHERS
ASSET(SUPERCOMPUTER, ZERDIA)	ASSET(FEATHERS, KARLA)
ATTACKS(GAGRACH, ZERDIA, MISSILES)	SEES(KARLA, HUNTER)
LACKS(MISSILES, SUPERCOMPUTER)	FOLLOWS(ATTACKS, SEES)
MISSES(MISSILES, ZERDIA)	ATTACKS(HUNTER, KARLA, CROSS-BOW)
CAUSES(LACKS, MISSES)	LACKS(CROSS-BOW, FEATHERS)
WANTS(GAGRACH, SUPERCOMPUTER)	MISSES(CROSS-BOW, KARLA)
REALIZES(ZERDIA, WANTS)	CAUSES(LACKS, MISSES)
FOLLOWS(REALIZES, MISSES)	WANTS(HUNTER, FEATHERS)
OFFERS(ZERDIA, SUPERCOMPUTER, GAGRACH)	REALIZES(KARLA, WANTS)
CAUSES(REALIZES, OFFERS)	FOLLOWS(REALIZES, MISSES)
OBTAINS(GAGRACH, SUPERCOMPUTER)	OFFERS(KARLA, FEATHERS, HUNTER)
CAUSES(OFFERS, OBTAINS)	CAUSES(REALIZES, OFFERS)
PLEASES(OBTAINS, GAGRACH)	OBTAINS(HUNTER, FEATHERS)
NOT ATTACK(GAGRACH, ZERDIA, MISSILES)	CAUSES(OFFERS, OBTAINS)
PROMISES(GAGRACH, ZERDIA, NOT ATTACK)	PLEASES(OBTAINS, HUNTER)
CAUSES(PLEASES, PROMISES)	NOT ATTACK(HUNTER, KARLA, CROSS-BOW)
	PROMISES(HUNTER, KARLA, NOT ATTACK)
	CAUSES(PLEASES, PROMISES)

correspondences remain uncertain, but when the influence of one-to-one correspondence is strong, competition drives correspondences from uncertainty to either certainly correspond or certainly not correspond.

The simulations of Rutherford's analogy and the "Karla the hawk" analogy suggest that appropriate values for the parameters relation-to-relation-inhibit-wt and feature-to-feature-inhibit-wt depend on the size of the representations being compared. Comparisons involving large representations require more inhibition between correspondences that taken together violate one-to-one correspondence. The relationship between scale and inhibition can be explained in terms of combinatorics. The constraint of one-to-one correspondence plays a critical role in reducing the number of potential mappings between representations. As described in Section 2.2, if each representation has n elements, then there are 2^{n^2} potential mappings between the two representations. However, the number of one-to-one mappings is $n!$. For example, if each representation has 5 elements, then there are over 10 million potential mappings, but only 120 one-to-one mappings. Thus, the larger the representations being compared, the more important the constraint of one-to-one correspondence becomes in reducing the number of potential mappings.

Chapter 6

Conclusions

6.1 General Discussion

Chapter 2 lays out twelve benchmark phenomena that must be accounted for by a model of similarity and analogy. As described in Chapter 5, Tuk-Tuk does a good job accounting for these phenomena. In addition, the behavioral study and simulations described in Section 5.1 distinguish Tuk-Tuk from other models in its ability to account for patterns of similarity ratings. Tuk-Tuk's successes underscore several points:

1. Comparisons involve determining correspondences between compared items. Because mapping is at the core of Tuk-Tuk's processing, Tuk-Tuk can account for phenomena that cannot be addressed by traditional models. For example, Tuk-Tuk captures the psychological distinction between MIPs, MOPs, alignable differences, and non-alignable differences. While previous models of analogy have illuminated constraints on mapping, these models are limited in their ability to account for a broader range of comparisons including similarity judgments. To a large extent, this limitation reflects a failure to fully capture the nature of the mapping process.

2. Correspondences are determined by a dynamic process of interactive activation among

feature, entity, and relation correspondences. Correspondences interact at each level of representation such that feature correspondences influence each other, entity correspondences influence each other, and relation correspondences influence each other. Correspondences also interact between levels of representation such that relation correspondences influence entity correspondences and vice versa, and entity correspondences influence feature correspondences and vice versa.

3. Correspondences are influenced by structural constraints such that structurally consistent correspondences excite each other and structurally inconsistent correspondences inhibit each other. As a result, correspondences are not only determined by semantic similarity. For example, dissimilar entities may be placed into correspondence if they play similar roles in matching systematic relational structures or if the correspondence is compatible with correspondences between other entities that are similar.

4. Comparisons utilize structured representations. Whereas traditional models are limited to spatial or feature-set representations, Tuk-Tuk operates over hierarchical and propositional representations that have interrelated and internally organized parts. Capturing the structure of mental representations is prerequisite to addressing structural constraints on the comparison process. For example, issues of structural consistency arise because entities are comprised of features. If two entities correspond, then structural consistency requires that their features correspond. Likewise, if two relations correspond, then structural consistency requires that their arguments correspond.

5. Comparisons highlight corresponding elements over noncorresponding elements. Tuk-Tuk accounts for domain-general regularities in which corresponding elements are more salient than noncorresponding elements. Matching and mismatching features and relations influence similarity more if they correspond than if they do not corre-

spond. Thus, the attention given to a particular element depends on the items being compared.

6. Comparisons are dynamic. Tuk-Tuk accounts for the development of correspondences and similarity over time. Comparisons are driven by semantic commonalities early in processing and reflect structural constraints over time. Early in processing, MIPs have the same influence as MOPs and alignable differences have the same influence as nonalignable differences. Over time, MIPs and alignable differences have a greater influence than MOPs and nonalignable differences.

Perhaps the most serious limitation of Tuk-Tuk is that its knowledge representations are hand-coded. This limitation is important because Tuk-Tuk is sensitive to the manner in which compared items are represented. For example, Section 5.3.11 describes simulations in which Tuk-Tuk generates either a relation-driven or a feature-driven mapping depending on differences in knowledge representation. Even relatively simple comparisons can depend on differences in knowledge representation. For example, the similarity between ABOVE(A, B) and ABOVE(C, D) is apparent to Tuk-Tuk, but the similarity between ABOVE(A, B) and BELOW(D, C) is not.

In fairness to Tuk-Tuk, this criticism applies broadly. No domain-general model of representation construction exists. While such a model seems far off, there have been several notable attempts to address the issue of representation construction. As described in Section 3.3.2, Copycat's interpretations of letter-string analogies are built up from representational primitives including letters, groups of letters, and relationships between letters. However, Forbus et al. (1998) lay bare Copycat's limitations:

> Copycat is unable to make correspondences between classes of statements that are not explicitly foreseen by its designers. Copycat cannot learn, because it cannot modify or extend these hand-coded representations that are essential to its operation. More fundamentally, it cannot capture what is perhaps the most

important creative aspect of analogy: the ability to align and map systems of knowledge from different domains. (p. 245)

Several models create postulated representations from obtained similarity data (Shepard, 1962b, 1962a, 1972; Shepard & Arabie, 1979; Tenenbaum, 1996; Navarro & Lee, 2003). However, these models are more descriptive than predictive. Lastly, Yan, Forbus, and Gentner (2003) propose a theory of rerepresentation in analogical mapping that divides the problem into detecting opportunities for rerepresentation, generating rerepresentation suggestions based on libraries of general methods, and controlling the rerepresentation process. While this theory is being developed with respect to SME, Tuk-Tuk could be extended in the same way.

6.2 Future Directions

Tuk-Tuk is capable of addressing a broad range of comparisons and speaks not only to the final state of comparisons, but also to the processing of comparisons over time. These qualities make Tuk-Tuk an ideal tool for advancing empirical investigations of the comparison process. The simulations described in this dissertation raise several interesting questions for future investigation:

1. Does systematic relational structure shared by compared items amplify the difference between corresponding and noncorresponding information in influencing similarity? In systematic comparisons, do MIPs increase similarity more and MOPs increase similarity less than in nonsystematic comparisons? Do alignable differences decrease similarity more and nonalignable differences decrease similarity less in systematic comparisons than in nonsystematic comparisons?

2. Is the time-course of the difference between MIPs and MOPs accelerated by systematicity? That is, are MIPs highlighted over MOPs earlier in systematic comparisons than in nonsystematic comparisons?

3. What are the time-courses of alignable and nonalignable differences? Do these time-courses mirror those of MIPs and MOPs?
4. Does adding matching systematic higher-order relational structure (e.g., causal relations) to an analogy speed or slow processing?
5. Do nonmonotonicities generalize to include analogies? That is, do matching relations that distract from dominant correspondences make an analogy more difficult to interpret and lead to lower ratings of analogousness or inferential soundness?
6. Are analogical nonmonotonicities prevalent throughout the time-course of processing?
7. Is one-to-one correspondence a soft constraint on the comparison process, or do people integrate similarity judgments and correspondences across multiple global interpretations, each of which is strictly one-to-one?

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This dissertation was typeset with LaTeX 2_ε[1] by the author.

[1] LaTeX 2_ε is an extension of LaTeX. LaTeX is a collection of macros for TeX. TeX is a trademark of the American Mathematical Society. The macros used in formatting this dissertation were written by Dinesh Das, Department of Computer Sciences, The University of Texas at Austin, and extended by Bert Kay, James A. Bednar, and Ayman El-Khashab.